

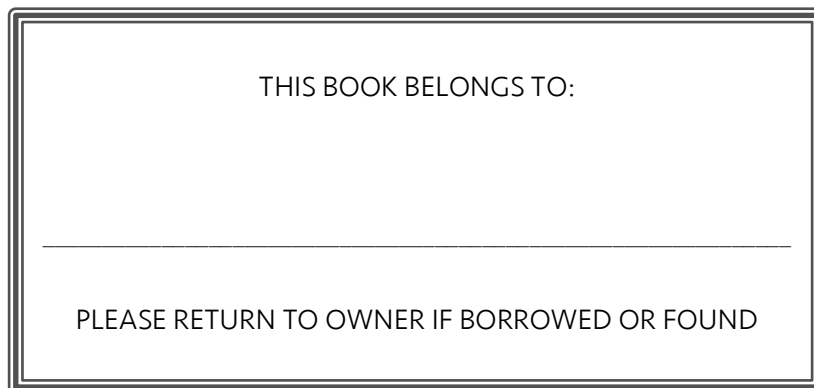
**SUMMIT  
MATH**

Learn at your **OWN** pace.

# ALGEBRA 1

second edition

# 1 PERCENTS & RATES



### **DEDICATION**

To Lauren, Chloe, Dawson and Teagan

### **ACKNOWLEDGEMENTS**

I started writing these books in 2013 to help my students learn better. I kept writing them because I received encouraging feedback from students, parents and teachers. Thank you to all who have used these books, pointed out my mistakes, and made suggestions along the way. Thank you to all of the students and parents who asked me to keep writing more books. Thank you to my family for supporting me through every step of this journey.

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# ***INTRODUCTION***

## **Learning math through Guided Discovery:**

A Guided Discovery learning experience is designed to help you experience a feeling of discovery as you learn each new topic.

## **Why this curriculum series is named Summit Math:**

Learning through Guided Discovery can be compared to climbing a mountain. Climbing and learning both require effort and persistence. In both activities, people naturally move at different paces, but they can reach the summit if they keep moving forward. Whether you race rapidly through these books or step slowly through each scenario, this curriculum is designed to keep advancing your learning until you reach the end of the book.

## **Guided Discovery Scenarios:**

The Guided Discovery Scenarios in this book are written and arranged to show you that new math concepts are related to previous concepts you have already learned. Try to fully understand each scenario before moving on to the next one. To do this, try the scenario on your own first, check your answer when you finish, and then fix any mistakes, if needed. Making mistakes and struggling are essential parts of the learning process.

## **Homework and Extra Practice Scenarios:**

After you complete the scenarios in each Guided Discovery section, you may think you know those topics well, but over time, you will forget what you have learned. Extra practice will help you develop better retention of each topic. Use the Homework and Extra Practice Scenarios to improve your understanding and to increase your ability to retain what you have learned.

## **The Answer Key:**

The Answer Key is included to promote learning. When you finish a scenario, you can get immediate feedback. When the Answer Key is not enough to help you fully understand a scenario, you should try to get additional guidance from another student or a teacher.

## **Star symbols:**

Scenarios marked with a star symbol ★ can be used to provide you with additional challenges. Star scenarios are like detours on a hiking trail. They take more time, but you may enjoy the experience. If you skip scenarios marked with a star, you will still learn the core concepts of the book.

## **To learn more about Summit Math and to see more resources:**

Visit [www.summitmathbooks.com](http://www.summitmathbooks.com).



# GUIDED DISCOVERY SCENARIOS

As you complete scenarios in this part of the book, follow the steps below.

## Step 1: Try the scenario.

Read through the scenario on your own or with other classmates. Examine the information carefully. Try to use what you already know to complete the scenario. Be willing to struggle.

## Step 2: Check the Answer Key.

When you look at the Answer Key, it will help you see if you fully understand the math concepts involved in that scenario. It may teach you something new. It may show you that you need guidance from someone else.

## Step 3: Fix your mistakes, if needed.

If there is something in the scenario that you do not fully understand, do something to help you understand it better. Go back through your work and try to find and fix your errors. Mistakes provide an opportunity to learn. If you need extra guidance, get help from another student or a teacher.

After Step 3, go to the next scenario and repeat this 3-step cycle.

## NEED EXTRA HELP? watch videos online

Teaching videos for every scenario in the Guided Discovery section of this book are available at [www.summitmathbooks.com/algebra-1-videos](http://www.summitmathbooks.com/algebra-1-videos).

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*Section 1*  
***INTRODUCTION TO  
PERCENTS***

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*GUIDED DISCOVERY SCENARIOS*

A percentage is one way to express a fraction. If you set a goal to read 10 pages in a book, after you have read 5 of them, you have finished 5 out of 10, or five-tenths of your goal. Using percentage language, you can say you have read 50 percent (50%) of your goal.

1. To learn the topics in this book, you should already be familiar with some percentage concepts. For example, you should know how to write a fraction as a percent.

Without a calculator, how can you convert  $\frac{1}{5}$  to a percent?

2. When it is difficult to convert a fraction to a percent, it is likely a better use of your time to use a calculator to do the conversion.

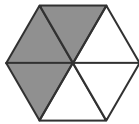
Using a calculator, how can you convert  $\frac{19}{40}$  to a percent?

3. What portion of the figure is shaded in each image below? Express this as a simplified fraction.

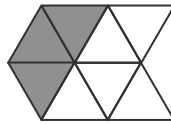
a.



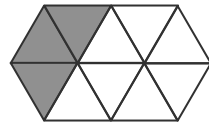
b.



c.



d.



4. Convert each of the previous fractions to percents.

5. When you calculate a percent of another number, like 10% of 200, you identify a specific fractional amount of that number. Some percents are easier to compute. For example, what is 50% of 10?



6. Some percentages are more challenging but can still be computed without a calculator if you are familiar with common percents.

a. How much money is 25% of \$4?

b. You bought 20 pounds of apples at an orchard and you used 75% of the apples to make applesauce. How many pounds of apples did you have left over?

Other percent calculations, like 35% of 43, or 12% of 112, require more time and can be done more quickly with a calculator. This brings up an important point, though. A calculator is only helpful if you input correct values.

7. For example, if you try to determine 50% of 10 by making a calculator compute  $50 \times 10$ , the calculator will give you a result of \_\_\_\_\_. Explain why this result is not reasonable.

8. If you can estimate percentages before you do calculations, you might notice your errors if you make mistakes. Try to estimate each value below. Do not use a calculator.

a. 48% of 12

b. 26% of 16

c. 99.1% of 20

A percent can also be more than 100%. For example, 300% of 20 means 3 sets of "100% of 20." Since 100% of 20 is exactly 20, then 300% of 20 is 3 sets of 20, which is 60.

9. Use a calculator to compute 364% of 45.

10. Sometimes it will be useful to write an expression like "22% of a number" in a smaller format, such as 22% of  $N$ , or even smaller as  $0.22N$ . The expression  $0.22N$  shows that you can compute 22% of  $N$  by multiplying the decimal form of the percent by the variable  $N$ . Write an expression that represents each of the following, using decimals instead of percents.

a. 50% of  $A$

b. 3% of  $B$

c. 275% of  $C$

d. 0.4% of  $D$

Now that you have reviewed some percentage topics, this book will build on these topics and guide you through a selection of scenarios that use percents to make comparisons and calculations.



*Section 2*  
***PERCENT CHANGES:  
INCREASES & DECREASES***

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11. A puffer fish has a circumference of 10 inches. When it feels threatened, it sucks in water and increases its circumference to 15 inches. To show a numerical change like this, you can compare the amount of the increase to the original amount. The fish grows by 5 inches, which is \_\_\_\_\_% of its original circumference. Using percents, the circumference grows by \_\_\_\_\_%.
12. If the fish continues to feel threatened, it expands its body more, until its circumference is 20 inches. Since the fish has now expanded by a total of 10 inches, it has increased its original circumference by \_\_\_\_\_%. This shows an interesting detail. If a number increases to become twice as large as its original value, it has increased by \_\_\_\_\_%.
13. If a number becomes three times as large as its initial value, by what percent has it increased?
14. If a number becomes one fourth of its original value, by what percent has it decreased?
15. Suppose that you measure your height one year and find that you are 50 inches tall. One year later, you measure again to check your growth and you find that your height has increased to 54 inches. In order to describe your growth as a percent increase you need to first consider your amount of growth, which is 4 inches.
- The amount of growth is a fractional amount of your original height, in this case  $\frac{4}{50}$ . This fraction can be simplified and written as the fraction \_\_\_\_\_.
  - Every fraction can be represented in decimal form. Use a calculator to write the value of  $\frac{2}{25}$  in its decimal form.
  - If you express 0.08 as a percent, it is \_\_\_\_\_%, which leads us back to the height scenario above. If your height increases from 50 inches to 54 inches, you can say that your height has increased by \_\_\_\_\_%.
16. Your height increases from 54 inches to 58 inches during the next year, another yearly increase of 4 inches. Does this represent an 8% increase again? If not, by what percent did your height increase if it changed from 54 to 58 inches?

*GUIDED DISCOVERY SCENARIOS*

The previous scenario involves challenging numbers. Return to simpler numbers for a moment in order to gain more familiarity with percentage changes.

17. A shirt initially costs \$50, but it goes on sale for \$40. Its price was decreased by \_\_\_\_\_%.
  
18. There are 80 students who are absent on Monday, and 100 students are absent on Tuesday. From Monday to Tuesday, the number of absences increased by \_\_\_\_\_%.
  
19. An \$80 pair of boots decreases by \$20 to sell for \$60. A pair of shoes is priced at \$60, but it only sells for \$40. Which item had its price reduced by a greater percent, the boots or the shoes?
  
20. If 40 people earned a passing score last year, and only 20 people earned a passing score this year, by what percent did the number of passing scores decrease?
  
21. If you owe \$20 to your friend today, and tomorrow you pay the entire amount that you owe, by what percent did you reduce the amount of money that you owed your friend?
  
22. When 100 increases to become 140, it might be easy for you to see that 100 has increased by 40 percent, because 100 is an easy number to work with. The number 100 is the foundation of percentages. The word percent comes from the phrase "per cent," which means "per 100." Try something more challenging to see if you understand the concept of percent change. If \$140 increases to become \$180, by what percent has it increased?
  
23. Fill in each blank below.
  - a. If 100 increases by 20%, its new value is \_\_\_\_\_.
  - b. If 100 decreases by 20%, its new value is \_\_\_\_\_.
  - c. If 80 increases by 35%, its new value is \_\_\_\_\_.
  - d. If 80 decreases by 35%, its new value is \_\_\_\_\_.

*Section 3*  
***WRITING EQUATIONS TO  
CALCULATE PERCENTS***

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In this section, you will learn how to write equations to solve percent scenarios.

35. Write each statement as an equation, but do not solve the equation.

- a. The product of a number and 5 is 20.
- b. The quotient of 100 and a number is 50.
- c. 6 is 7 less than 3 times a number.

36. Write each statement as an equation, but do not solve the equation.

- a. A number is 3 more than 10.
- b. A number is 6 less than 4.
- c. 3 is 4 more than a number.
- d. A number increased by 5 is 15.

The previous two scenarios do not involve percentages, but they are intended to help you think about writing mathematical statements as equations. The next scenarios will focus only on percentages.

37. Write each statement as an equation. Do not solve the equation.

- a. 50% of a number is 45.
- b. A number is 24% of 80.
- c. 15 is a percentage of 20.

38. Consider the question, "What number is 45% of 18?" It is difficult to mentally compute this, but it is easier to write it out as a statement and convert that statement into an equation: "A number is 45% of 18." Write this statement as an equation.

# Section 10

# ANSWER KEY

1.	Other options exist, but one way is to change the denominator to 100. This makes it easy to state the fraction as a percent. $\frac{1}{5} \cdot \frac{20}{20} \rightarrow \frac{20}{100}$ "20 hundredths" is 20 per 100 which has the same meaning as 20%.
2.	Find the decimal form of the fraction. In the calculator, divide 19 by 40. To do this, input $19 \div 40$ or $19/40$ . As a decimal, $19/40$ is 0.475, which is 475 thousandths or 47.5 hundredths. 47.5 per 100 is 47.5 percent (47.5%). You may have learned to move the decimal 2 places to the right to convert a decimal to a percent, and this is true but it is important to know why that works.
3.	a. $\frac{3}{4}$ b. $\frac{3}{6} \rightarrow \frac{1}{2}$ c. $\frac{3}{8}$ d. $\frac{3}{10}$
4.	a. 75%    b. 50%    c. 37.5%    d. 30%
5.	One-half of 10, which is 5.
6.	a. \$1    b. 5 pounds (75% of 20 is 15)
7.	500; 50% of a number is one-half of that number. One-half of 10 is just 5, so a result of 500 is not reasonable.
8.	a. $\approx 6$ (about 50% of 12) b. $\approx 4$ (about 25% of 16) c. $\approx 20$ (about 100% of 20)
9.	$3.64 \times 45 \rightarrow 163.8$
10.	a. 0.5A    b. 0.03B    c. 2.75C    d. 0.004D
11.	50%
12.	100%
13.	200%
14.	75%
15.	a. $\frac{2}{25}$ b. 0.08    c. 8
16.	Approx. 7.4% $\left( \frac{4}{54} \text{ is } 0.074 \text{ in decimal form} \right)$
17.	20%
18.	25%
19.	the shoes $\left( 33\frac{1}{3}\% \text{ vs. } 25\% \right)$
20.	50%
21.	100%
22.	$\frac{40}{140} \rightarrow 0.286 \rightarrow \text{approx. } 28.6\%$
23.	a. 120    b. 80    c. 108    d. 52
24.	a. $E + 0.5E$ b. $F - 0.25F$ c. $G + 0.05G$ d. $H - 0.15H$
25.	a. 1.5E    b. 0.75F    c. 1.05G    d. 0.85H
26.	a. 1.35X    b. 0.81X    c. 2X    d. 0
27.	a. 170    b. 75
28.	a. 20% (increase)    b. 10% (decrease) c. 80% (decrease)
29.	a. 120% (increase)    b. 200% (increase) c. 350% (increase)
30.	72% (decrease)
31.	a. increase 64%    b. increase 150% c. decrease 1%    d. increase 400%
32.	$\frac{259}{490} \rightarrow 0.529 \rightarrow \text{approx. } 52.9\%$
33.	$\frac{253}{30} \rightarrow 8.43\bar{3} \rightarrow 843\frac{1}{3}\%$
34.	Between June and July (20.7% increase)
35.	a. $5n = 20$ b. $100 \div n = 50$ c. $6 = 3n - 7$
36.	a. $n = 10 + 3$ b. $n = 4 - 6$ c. $3 = n + 4$ d. $n + 5 = 15$
37.	a. $0.5n = 45$ b. $n = 0.24(80)$ c. $15 = \frac{n}{100}(20)$ or $15 = n(20) \rightarrow$ if $n$ is the decimal form of the percentage.
38.	$n = 0.45(18)$
39.	a. $X = 0.12(30)$ b. $0.12(30) = X$ c. $45 = 0.2X$ d. $0.2X = 45$
40.	a,b. 3.6    c,d. 225
41.	a. $18 = \frac{x}{100}(24)$ b. $\frac{x}{100}(24) = 18$ c. $36 = \frac{x}{100}(15)$ d. $\frac{x}{100}(15) = 36$
42.	a, b. 75%    c, d. 240%
43.	a. 45% of 200 is $B \rightarrow 0.45(200) = B$ b. 65% of $F$ is 70 $\rightarrow 0.65F = 70$