

Algebra 1 Series Book 5 Homework Guide

# A NOTE TO PARENTS AND TEACHERS:

Each book in the Summit Math Algebra 1 Series has 2 parts. The first half of the book is the Guided Discovery Scenarios. The second half of the book is the Homework & Extra Practice Scenarios. Each section has a separate Answer Key. If the Answer Key does not provide enough guidance, you can access more information about how to solve each scenario using the resources listed below.

## 1. GUIDED DISCOVERY SCENARIOS

If you would like to get step-by-step guidance for the Guided Discovery Scenarios in each Algebra 1 book, you can subscribe to the Algebra 1 Videos for \$9/month or \$60/year (\$5/mo.). With a subscription, you can access videos for every book in the Series. The videos show you how to solve each scenario in the Guided Discovery Scenarios section of the Algebra 1 books. You can find out more about these videos at www.summitmathbooks.com/algebra-1-videos.

## 2. HOMEWORK & EXTRA PRACTICE SCENARIOS

If you would like to get step-by-step guidance for the Homework & Extra Practice Scenarios in the book, you can use this Homework Guide. It provides more detailed guidance for solving the Homework & Extra Practice Scenarios in Book 5 of the Algebra 1 Series. Some scenarios are not included. If you would like something included in this Homework Guide, please email the author and explain which scenario(s) you would like to see included or which scenario(s) you would like more guidance for in this Homework Guide.

# **ANSWER KEY**

2a.  $32 \rightarrow \text{ write } 32 \text{ as } 16\cdot 2$   $16\cdot 2 \rightarrow \text{ write } 16 \text{ as } 8\cdot 2$   $8\cdot 2\cdot 2 \rightarrow \text{ write } 8 \text{ as } 4\cdot 2$   $4\cdot 2\cdot 2\cdot 2 \rightarrow \text{ write } 4 \text{ as } 2\cdot 2$  $2\cdot 2\cdot 2\cdot 2\cdot 2 \rightarrow 2^5$  2b.  $36 \rightarrow \text{ write } 36 \text{ as } 9.4$   $9.4 \rightarrow \text{ write } 9 \text{ as } 3.3$   $3.3.4 \rightarrow \text{ write } 4 \text{ as } 2.2$  $3.3.2.2 \rightarrow 3^2.2^2$ 

За.	3b.	Зс.
48 $\rightarrow$ write 48 as 8.6	56 $\rightarrow$ write 56 as 8.7	$63 \rightarrow \text{write } 63 \text{ as } 9.7$
8.6 $\rightarrow$ write 6 as 3.2	8.7 $\rightarrow$ write 8 as 4.2	9.7 $\rightarrow$ write 9 as 3.3
$8 \cdot 3 \cdot 2 \rightarrow \text{write } 8 \text{ as } 4 \cdot 2$	$4.2.7 \rightarrow \text{write } 4 \text{ as } 2.2$	$3\cdot 3\cdot 7 \rightarrow 3^2 \cdot 7$
$4 \cdot 2 \cdot 3 \cdot 2 \rightarrow \text{write } 4 \text{ as } 2 \cdot 2$	$2 \cdot 2 \cdot 2 \cdot 7 \rightarrow 2^3 \cdot 7$	
$2 \cdot 2 \cdot 2 \cdot 3 \cdot 2 \rightarrow \text{group the } 2's$		
$2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \rightarrow 2^4 \cdot 3$		

## 8a.

The expression  $4B^2 + 24B$  is the sum of 2 terms:  $4B^2$  and 24B. These 2 terms have the following monomial factors in common: 2, 4, B, 2B and 4B. The greatest factor they have in common is 4B.  $4B^2$  is  $4B \cdot B$  and 24B is  $4B \cdot 6$ . If you write the expression  $4B^2 + 24B$  as the product of its <u>greatest</u> monomial factor, you can also write the expression  $4B^2 + 24B$  in any of the forms below:

 $2(2B^2 + 12B)$   $4(B^2 + 6B)$  B(4B + 24) 2B(2B + 12)

As you can see, when the 2 terms in a binomial have many factors in common, there are many ways to write the binomial as the product of a monomial and a binomial. This is why it is typical to choose the monomial that is the greatest common factor.

8b.

The expression  $32x^2 - 24x$  is the difference of 2 terms:  $32x^2$  and 24x. These 2 terms have the following monomial factors in common: 2, 4, 8, x, 2x, 4x and 8x. The greatest factor they have in common is 8X.  $32x^2$  is  $8x \cdot 4x$  and 24x is  $8x \cdot 3$ . If you write the expression  $32x^2 - 24x$  as the product of its greatest monomial factor and a binomial, it can be written as 8x(4x - 3). If you choose a different monomial factor, you can also write the expression  $32x^2 - 24x$  in any of the forms below:

$$2(16x^2 - 12x) = 4(8x^2 - 6x) = 8(4x^2 - 3x) = x(32x - 24) = 2x(16x - 12) = 4x(8x - 6)$$

As you can see, when the 2 terms in a binomial have many factors in common, there are many ways to write the binomial as the product of a monomial and a binomial. This is why it is typical to choose the monomial that is the greatest common factor.

When you factor a trinomial that looks like  $x^2 + bx + c$ , the factors are  $(x + \_)(x + \_)$ . Instead of empty blanks, suppose you write the factors as (x + D)(x + E). The correct factors of  $x^2 + bx + c$  are (x + D)(x + E) if  $D \cdot E = c$  and D + E = b.

Let's apply this to  $x^2 + 5x + 6$ . The factored form of this trinomial is (x + 2)(x + 3) because  $2 \cdot 3 = 6$  and 2 + 3 = 5. Check your factors by multiplying them to confirm you get the original trinomial.

## 21b.

21a.

When you factor a trinomial that looks like  $x^2 + bx + c$ , the factors are  $(x + \_)(x + \_)$ . Instead of empty blanks, suppose you write the factors as (x + D)(x + E). The correct factors of  $x^2 + bx + c$  are (x + D)(x + E) if  $D \cdot E = c$  and D + E = b.

Let's apply this to  $x^2 + 7x + 12$ . The factored form of this trinomial is (x + 3)(x + 4) because  $3 \cdot 4 = 12$  and 3 + 4 = 7. Check your factors by multiplying them to confirm you get the original trinomial.

## 22a.

The factored form of  $x^2 + 10x + 16$  is (x + 8)(x + 2) because  $8 \cdot 2 = 16$  and 8 + 2 = 10. Check your factors by multiplying them to confirm you get the original trinomial.

## 22b.

It may help to think about  $x^2 - x - 6$  as  $x^2 - 1x - 6$ . The factored form is (x - 3)(x + 2) because  $-3 \cdot 2 = -6$  and -3 + 2 = -1. Check your factors by multiplying them to confirm you get the original trinomial.

## 22a.

The factored form of  $x^2 - 4x - 12$  is (x - 6)(x + 2) because  $-6 \cdot 2 = -12$  and -6 + 2 = -4. Check your factors by multiplying them to confirm you get the original trinomial.

## 22b.

The factored form of  $x^2 + 15x - 16$  is (x + 16)(x - 1) because  $16 \cdot -1 = -16$  and 16 + (-1) = 15. Check your factors by multiplying them to confirm you get the original trinomial.

## 26a.

The factored form of  $x^2 + 12x + 11$  is (x + 11)(x + 1) because  $11 \cdot 1 = 11$  and 11 + 1 = 12. Check your binomial factors by multiplying them.

## 26b.

It may help to think about  $x^2 - 36$  as  $x^2 + 0x - 36$ . The factored form is (x + 6)(x - 6) because  $6 \cdot - 6 = -36$  and 6 + (-6) = 0. Check your binomial factors by multiplying them.

## 26c.

The factored form of  $x^2 + 20xy + 100y^2$  is (x + 10y)(x + 10y) because  $10y \cdot 10y = 100y^2$  and 10 + 10 = 20. Check your binomial factors by multiplying them.

32.

The first term in the trinomial  $-x^2 + 14x - 24$  is negative. When the first term is  $-x^2$ , it is often a good idea to factor out a -1 and write the trinomial as  $-(x^2 - 14x + 24)$ . Now you can focus on factoring the trinomial inside the parentheses. The factored form of  $x^2 - 14x + 24$  is (x - 12)(x - 2) because  $-12 \cdot -2 = 24$  and -12 + (-2) = -14. Check your binomial factors by multiplying them.

The factored form of the trinomial  $-x^2 + 14x - 24$  is -(x - 12)(x - 2).

# 33a.

The factored form of  $x^2 - 33x + 200$  is (x - 25)(x - 8) because  $-25 \cdot - 8 = 200$  and -25 + (-8) = -33. Check your binomial factors by multiplying them.

# 33b.

The factored form of  $x^2 + 73x - 150$  is (x - 2)(x + 75) because -2.75 = -150 and -2 + 75 = 73. Check your binomial factors by multiplying them.

## 33c.

The factored form of  $x^2 + 29x + 100$  is (x + 4)(x + 25) because  $4 \cdot 25 = 100$  and 4 + 25 = 29. Check your binomial factors by multiplying them.

## 34a.

The factored form of  $x^2 + 16x + 39$  is (x + 13)(x + 3) because  $13 \cdot 3 = 39$  and 13 + 3 = 16. Check your binomial factors by multiplying them.

## 34b.

The factored form of  $x^2 - 8x + 7$  is (x - 1)(x - 7) because  $-1 \cdot -7 = 7$  and -1 + (-7) = -8. Check your binomial factors by multiplying them.

## 34c.

The factored form of  $x^2 - 5x - 126$  is (x + 9)(x - 14) because  $9 \cdot (-14) = -126$  and 9 + (-14) = -5. Check your binomial factors by multiplying them.

## 35a.

The factored form of  $x^2 + 7x + 6$  is (x + 1)(x + 6) because  $1 \cdot 6 = 6$  and 1 + 6 = 7. Check your binomial factors by multiplying them.

## 35b.

The factored form of  $x^2 - 7x + 6$  is (x - 1)(x - 6) because  $-1 \cdot - 6 = 6$  and -1 + (-6) = -7. Check your binomial factors by multiplying them.

## 36a.

It may help to think about  $x^2 - x - 6$  as  $x^2 - 1x - 6$ . The factored form is (x - 3)(x + 2) because  $-3 \cdot 2 = -6$  and -3 + 2 = -1. Check your binomial factors by multiplying them.

## 36b.

It may help to think about  $x^2 + x - 6$  as  $x^2 + 1x - 6$ . The factored form is (x + 3)(x - 2) because  $3 \cdot (-2) = -6$  and 3 + (-2) = 1. Check your binomial factors by multiplying them.

HOMEWORK & EXTRA PRACTICE SCENARIOS 38a. The factored form of  $x^2 - 5x - 6$  is (x - 6)(x + 1) because  $-6 \cdot 1 = -6$  and -6 + 1 = -5. Check your factors by multiplying them to confirm you get the original trinomial. 38b.  $20x^2 - 100x - 120 \rightarrow$  factor out the greatest common factor of 20  $20(x^2 - 5x - 6) \rightarrow$  write the trinomial as the product of 2 binomials 20(x-6)(x+1)39a.  $20x^2 - 100x - 120 \rightarrow$  factor out the greatest common factor of  $2x^4$  $2x^4(x^2 - 5x - 6) \rightarrow$  factor the trinomial  $2x^4(x-6)(x+1)$ 39b.  $-3x^2 + 15x + 18 \rightarrow$  factor out the greatest common factor of -3 $-3(x^2 - 5x - 6) \rightarrow$  factor the trinomial -3(x-6)(x+1)40a.  $-x^3 + 5x^2 + 6x \rightarrow$  factor out the greatest common factor of -x $-x(x^2 - 5x - 6) \rightarrow$  factor the trinomial -x(x-6)(x+1)40b.  $3x^2 + 18x - 21 \rightarrow$  factor out the greatest common factor of 3  $3(x^2 + 6x - 7) \rightarrow$  factor the trinomial 3(x+7)(x-1)41a.  $-5x^2 + 40x - 60 \rightarrow$  factor out the greatest common factor of -5 $-5(x^2 - 8x + 12) \rightarrow$  factor the trinomial -5(x-6)(x-2)41b. When a trinomial looks like  $x^2 + bx + c$ , the factors are (x + )(x + ). This trinomial starts with  $2x^2$ , so the factored form is (2x + )(x + )(x + ). This type of trinomial may take more time to factor because one binomial factor starts with 2x and

Suppose the factored form of  $2x^2 - 7x - 9$  as (2x + A)(x + B). The product of A and B must be -9 so there are 4 possible options.

that 2x will affect the product of the binomials when you multiply them to see if they are correct.

#1: A = 9, B = -1 #2: A = -9, B = 1 #3: A = 3, B = -3 #3: A = -3, B = 3

Given these 4 possible options, there are 4 possible factored forms:

#1: (2x + 9)(x - 1) #2: (2x - 9)(x + 1) #3: (2x + 3)(x - 3) #4: (2x - 3)(x + 3)

41b. (continued)

If you multiply the binomials together, each of the 4 factored forms will have the same first and third term. Each of the products is  $2x^2 \dots -9$ . To find the correct factors, multiply the outer and inner terms to see which product creates a middle term of -7x. For example, if the factored form is (2x + 9)(x - 1), the outer terms are 2x and -1. The inner terms are +9 and x. The product of the outer terms is -2x and the product of the inner terms is +9x.

The middle term of each product is shown below:

#1:(2x+9)(x-1)	#2:(2x-9)(x+1)	#3:(2x+3)(x-3)	#4: (2x - 3)(x + 3)
-2x + 9x	+2x - 9x	-6x + 3x	+6x - 3x
+7x	—7x	—3x	+3x

The correct factored form of  $2x^2 - 7x - 9$  is option #2, which is (2x - 9)(x + 1).

44a.

Multiply the outer and inner terms of (2x + 7)(x - 1) to find the middle term of the trinomial  $2x^2 + x - 9$ . In the expression (2x + 7)(x - 1), the outer terms are 2x and -1. The inner terms are +7 and x. The product of the outer terms is -2x and the product of the inner terms is +7x. When you combine -2x + 7x, the middle term is +5x. The missing coefficient in  $2x^2 + x - 9$  is 5.

## 44b.

Multiply the outer and inner terms of (3x + 2)(x - 5) to find the middle term of the trinomial  $3x^2 + x - 10$ . In the expression (3x + 2)(x - 5), the outer terms are 3x and -5. The inner terms are +2 and x. The product of the outer terms is -15x and the product of the inner terms is +2x. When you combine -15x + 2x, the middle term is -13x. The missing coefficient in  $3x^2 + x - 10$  is -13.

## 44c.

Multiply the outer and inner terms of (2x - 1)(2x + 7) to find the middle term of the trinomial  $4x^2 + x - 7$ . In the expression (2x - 1)(2x + 7), the outer terms are 2x and +7. The inner terms are -1 and 2x. The product of the outer terms is +14x and the product of the inner terms is -2x. When you combine +14x - 2x, the middle term is +12x. The missing coefficient in  $4x^2 + x - 7$  is 12.

## 45a.

The factored form of  $3x^2 - 5x - 2$  is  $(\_\_ + 1)(\_\_ - 2)$ . The product of the 2<sup>nd</sup> term in each binomial is the third term in the original trinomial: -2. The product of the 1<sup>st</sup> term in each binomial is the first term in the original trinomial:  $3x^2$ . There are 2 possible options.

#1: (3x + 1)(x - 2) #2: (x + 1)(3x - 2)

Multiply the outer and inner terms to see which factored form produces a trinomial with a middle term of -5x.

#1: 
$$(3x + 1)(x - 2)$$
#2:  $(x + 1)(3x - 2)$  $-6x + 1x$  $-2x + 3x$  $-5x$  $+1x$ 

The correct factored form of  $3x^2 - 5x - 2$  is option #1, which is (3x + 1)(x - 2).

45b.

The factored form of  $5x^2 - 13x - 6$  is  $(\_ - 3)(\_ + 2)$ . The product of the 2<sup>nd</sup> term in each binomial is the third term in the original trinomial: -6. The product of the 1<sup>st</sup> term in each binomial is the first term in the original trinomial:  $5x^2$ . There are 2 possible options.

#1: 
$$(5x - 3)(x + 2)$$
 #2:  $(x - 3)(5x + 2)$ 

Multiply the outer and inner terms to see which factored form produces a trinomial with a middle term of -13x.

#1: 
$$(5x - 3)(x + 2)$$
#2:  $(x - 3)(5x + 2)$  $+10x - 3x$  $+2x - 15x$  $+7x$  $-13x$ 

The correct factored form of  $5x^2 - 13x - 6$  is option #2, which is (x - 3)(5x + 2).

## 45c.

The factored form of  $6x^2 - 7x - 3$  is  $(\_\_ + 1)(\_\_ - 3)$ . The product of the 2<sup>nd</sup> term in each binomial is the third term in the original trinomial: -3. The product of the 1<sup>st</sup> term in each binomial is the first term in the original trinomial:  $6x^2$ . The factors of  $6x^2$  could be 6x and 1x or 3x and 2x so there are 4 possible options.

#1: 
$$(6x + 1)(x - 3)$$
 #2:  $(x + 1)(6x - 3)$  #3:  $(3x + 1)(2x - 3)$  #4:  $(2x + 1)(3x - 3)$ 

Multiply the outer and inner terms to see which factored form produces a trinomial with a middle term of -7x.

#1: (6x + 1)(x - 3)#2: (x + 1)(6x - 3)#3: (3x + 1)(2x - 3)#4: (2x + 1)(3x - 3)-18x + 1x-3x + 6x-9x + 2x-6x + 3x-17x+3x-7x-3x

The correct factored form of  $6x^2 - 7x - 3$  is option #3, which is (3x + 1)(2x - 3).

46a.

Multiply the outer and inner terms of (3x - 8)(x + 3) to find the middle term of the trinomial  $3x^2 + Ux - 24$ . In the expression (3x - 8)(x + 3), the outer terms are 3x and +3. The inner terms are -8 and x. The product of the outer terms is +9x and the product of the inner terms is -8x. When you combine +9x - 8x, the middle term is +1x. The value of U is 1 in  $3x^2 + Ux - 24$ .

46b.

Multiply the outer and inner terms of (x + 5)(5x - 3) to find the middle term of the trinomial  $5x^2 + Ux - 15$ . In the expression (x + 5)(5x - 3), the outer terms are x and -3. The inner terms are +5 and 5x. The product of the outer terms is -3x and the product of the inner terms is +25x. When you combine -3x + 25x, the middle term is +22x. The value of U is 22 in  $5x^2 + Ux - 15$ .

## 46c.

Multiply the outer and inner terms of (6x + 1)(2x - 3) to find the middle term of the trinomial  $12x^2 + Ux - 3$ . In the expression (6x + 1)(2x - 3), the outer terms are 6x and -3. The inner terms are +1 and 2x. The product of the outer terms is -18x and the product of the inner terms is +2x. When you combine -18x + 2x, the middle term is -16x. The value of U is -16 in  $12x^2 + Ux - 3$ .

summitmathbooks.com

48a.

The factored form of  $5x^2 - 6x + 1$  is (5x + A)(x + B), since the only factors of  $5x^2$  are 5x and x. The product of A and B must be + 1 so there are 2 possible options.

#1: A = -1, B = -1 #2: A = 1, B = 1

Given these 2 possible options, there are 2 possible factored forms:

$$#1: (5x + 1)(x + 1) = #2: (5x - 1)(x - 1)$$

If you multiply the binomials together, each of the 2 factored forms will have the same first and third term. Each of the products is  $5x^2 \dots + 1$ . To find the correct factors, multiply the outer and inner terms to see which product creates a middle term of -6x. For example, if the factored form is (5x + 1)(x + 1), the outer terms are 5x and + 1. The inner terms are + 1 and x. The product of the outer terms is +5x and the product of the inner terms is +x.

The middle term of each product is shown below:

 $\begin{array}{rcl}
\#1: (5x + 1)(x + 1) & \#2: (5x - 1)(x - 1) \\
+5x + x & -5x - x \\
+6x & -6x
\end{array}$ 

The correct factored form of  $5x^2 - 6x + 1$  is option #2: (5x - 1)(x - 1).

48b.

The factored form of  $2x^2 + 9x + 7$  is (2x + A)(x + B), since the only factors of  $2x^2$  are 2x and x. The product of A and B must be + 7 so there are 2 possible options.

Given these 2 possible options, there are 2 possible factored forms:

#1: 
$$(2x + 7)(x + 1)$$
 #2:  $(2x + 1)(x + 7)$  #3:  $(2x - 7)(x - 1)$  #4:  $(2x - 1)(x - 7)$ 

The middle term of each product is shown below:

Option #1 has the correct middle term. The factored form of  $2x^2 + 9x + 7$  is (2x + 7)(x + 1).

48c.

It is probably easier to factor  $-x^2 - 2x + 3$  if it has a positive leading coefficient. To make the leading coefficient positive, factor out a -1 and write the expression as  $-(x^2 + 2x - 3)$ . Now you can factor the trinomial inside parentheses. Since the leading term is  $x^2$ , the factored form is (x + A)(x + B) if  $A \cdot B = -3$  and A + B = +2. This will be true if A = 3 and B = -1 so the factored form of  $x^2 + 2x - 3$  is (x + 3)(x - 1).

Since the original expression is  $-x^2 - 2x + 3$ , its factored form is -(x + 3)(x - 1).

49a.  $4x^2 - 7 - 3x \rightarrow$  write the trinomial in Standard Form  $4x^2 - 3x - 7$ (4x - 7)(x + 1)49b.  $3 + 2x^2 + 7x \rightarrow$  write the trinomial in Standard Form  $2x^2 + 7x + 3$ (2x + 1)(x + 3)49c.  $-3x + 4 - x^2 \rightarrow$  write the trinomial in Standard Form  $-x^2 - 3x + 4$  $-(x^2 + 3x - 4)$ -(x+4)(x-1)50a.  $5x^2 - 6 + 7x \rightarrow$  write the trinomial in Standard Form  $5x^2 + 7x - 6$ (5x - 3)(x + 2)50b.  $6x^2 - 17x + 10$ (6x - 5)(x - 2)50c.  $-x^{2} + 12x - 36$  $-(x^2 - 12x + 36)$ -(x-6)(x-6)51a.  $15 - 11x + 2x^2 \rightarrow$  write the trinomial in Standard Form  $2x^2 - 11x + 15$ (2x-5)(x-3)51b.  $30 - 22x + 4x^2 \rightarrow$  write the trinomial in Standard Form  $4x^2 - 22x + 30 \rightarrow$  factor out the greatest common factor of 2  $2(2x^2 - 11x + 15) \rightarrow$  factor the trinomial in parentheses: it is the same as the one in 51a. 2(2x-5)(x-3)

52a.  $-15x + 11x^2 - 2x^3 \rightarrow \text{ write the trinomial in Standard Form}$   $-2x^3 + 11x^2 - 15x \rightarrow \text{ factor out the greatest common factor of } -x$   $-x(2x^2 - 11x + 15) \rightarrow \text{ factor the trinomial in parentheses: it is the same as the one in 51a.}$ -x(2x - 5)(x - 3)

## 52b.

 $20x^2 - 110x + 150 \rightarrow$  factor out the greatest common factor of 10  $10(2x^2 - 11x + 15) \rightarrow$  factor the trinomial in parentheses: it is the same as the one in 51a. 10(2x - 5)(x - 3)

## 53.

 $-6x^2 + 33x - 45 \rightarrow$  factor out the greatest common factor of -3 $-3(2x^2 - 11x + 15) \rightarrow$  factor the trinomial in parentheses: it is the same as the one in 51a. -3(2x - 5)(x - 3)

## 54.

If you simplify the product (2x - 1)(2x - 6), the middle term is -12x - 2x, which is -14x. That is different than the middle term of the trinomial  $4x^2 - 12x + 6$ , so (2x - 1)(2x - 6) is not the factored form of  $4x^2 - 12x + 6$ .

## 55a. x<sup>2</sup> — 49

(x - 7)(x + 7)

If you write this expression as a trinomial, it is  $x^2 + 0x - 49$ . To make the binomials multiply together to form a middle term of 0x, the binomials must have the same 2 terms, with one important difference. If one binomial is Ax + B, the other binomial is Ax – B. The product of these two binomials has a middle term of 0x.

## 55b.

 $4x^2 - 25y^2$ (2x + 5y)(2x - 5y)

If you write this expression as a trinomial, it is  $4x^2 + 0xy - 25y^2$ . To make the binomials multiply together to form a middle term of 0xy, the binomials must have the same 2 terms, with one important difference. If one binomial is Ax + By, the other binomial is Ax - By. The product of these two binomials has a middle term of 0xy.

## 56a.

The factored form of  $x^2 + 0x - 36$  is (x + 6)(x - 6). The product of the outer terms is -6x and the product of the inner terms is +6x. The sum of these terms is -6x + 6x, which combines to form 0x.

## 56b.

The factored form of  $x^2 + 0x - 36$  is (x + 6)(x - 6). The product of the outer terms is -6x and the product of the inner terms is +6x. The sum of these terms is -6x + 6x, which combines to form 0x.

## 56c.

The factored form of  $64 + 0x - x^2$  is (8 + x)(8 - x). The product of the outer terms is -8x and the product of the inner terms is +8x. The sum of these terms is -8x + 8x, which combines to form 0x.

## 57a.

The factored form of  $81 + 0x - x^2$  is (9 + x)(9 - x). The product of the outer terms is -9x and the product of the inner terms is +9x. The sum of these terms is -9x + 9x, which combines to form 0x.

## 57b.

The factored form of  $x^2 - 100$  is (x + 10)(x - 10). The product of the outer terms is -10x and the product of the inner terms is +10x. The sum of these terms is -10x + 10x, which combines to form 0x. When a term has a coefficient of 0, it does not need to be written. The expression  $x^2 + 0x - 100$  can be written as  $x^2 - 100$ .

## 57c.

The factored form of  $121 - x^2$  is (11 + x)(11 - x). The product of the outer terms is -11x and the product of the inner terms is +11x. The sum of these terms is -11x + 11x, which combines to form 0x. When a term has a coefficient of 0, it does not need to be written. The expression  $121 + 0x - x^2$  can be written as  $121 - x^2$ .

## 58a.

This scenario is written to remind you that you should start by looking for a greatest common factor, GCF, when you factor polynomials. The two terms in this expression have a common factor of 3, but the leading term is negative. When the leading term is negative, it is probably better to factor out a GCF of -3 because the expression inside parentheses will start with a positive term.

 $-3x^2 + 27 \rightarrow$  factor out a GCF of -3 $-3(x^2 - 9) \rightarrow$  factor the binomial in parentheses -3(x + 3)(x - 3)

## 58b.

This scenario is written to remind you that you should start by looking for a greatest common factor, GCF, when you factor polynomials. The two terms in this expression have a common factor of  $x^2$ , but the leading term is negative. When the leading term is negative, it is probably better to factor out a GCF of  $-x^2$  because the expression inside parentheses will start with a positive term.  $-x^4 + x^2 \rightarrow \text{ factor out a GCF of } -x^2$ 

 $-x^{2}(x^{2}-1) \rightarrow$  factor the binomial in parentheses  $-x^{2}(x+1)(x-1)$ 

59a.

Start by looking for a greatest common factor, GCF, when you factor polynomials. The two terms in this expression have a common factor of 6.

 $6x^2 - 24 \rightarrow \text{factor out a GCF of 6}$  $6(x^2 - 4) \rightarrow \text{factor the binomial in parentheses}$ 6(x + 2)(x - 2)

59b.

Start by looking for a greatest common factor, GCF, when you factor polynomials. The two terms in this expression have a common factor of 6.

 $24 - 6x^2 \rightarrow \text{factor out a GCF of 6}$   $6(4 - x^2) \rightarrow \text{factor the binomial in parentheses}$ 6(2 + x)(2 - x)

If you rewrite the initial expression as  $-6x^2 + 24$ , its factored form would be -6(x + 2)(x - 2).

60a.  $-5x^2 + 45 \rightarrow$  factor out a GCF of -5 $-5(x^2 - 9) \rightarrow$  factor the binomial in parentheses -5(x+3)(x-3)60b.  $810 - 10x^2 \rightarrow$  factor out a GCF of 10  $10(81 - x^2) \rightarrow$  factor the binomial in parentheses 10(9 + x)(9 - x)If you rewrite the initial expression as  $-10x^2 + 810$ , its factored form would be -10(x + 9)(x - 9). 61a.  $64x - x^3 \rightarrow \text{factor out a GCF of } x$  $x(64 - x^2) \rightarrow$  factor the binomial in parentheses x(8+x)(8-x)If you rewrite the initial expression as  $-x^3 + 64x$ , its factored form would be -x(x + 8)(x - 8). 61b.  $-2x^3 + 98x \rightarrow$  factor out a GCF of -2x $-2x(x^2 - 49) \rightarrow$  factor the binomial in parentheses -2x(x+7)(x-7)63.  $2x^2 - 32 \rightarrow$  factor out a GCF of 2  $2(x^2 - 16) \rightarrow$  factor the binomial in parentheses 2(x+4)(x-4)64.  $x^4 - 16 \rightarrow$  This binomial is a difference of squares so it can be factored  $(x^2 + 4)(x^2 - 4) \rightarrow x^2 + 4$  is not factorable, but  $x^2 - 4$  is a difference of squares so it is factorable  $(x^{2}+4)(x+2)(x-2)$ 65.  $16x^4 - 1 \rightarrow$  This binomial is a difference of squares so it can be factored  $(4x^2 + 1)(4x^2 - 1) \rightarrow 4x^2 + 1$  is not factorable, but  $4x^2 - 1$  is a difference of squares so it is factorable  $(4x^2 + 1)(2x + 1)(2x - 1)$ 71b.  $12x - 3x^2 - 12 \rightarrow$  rearrange the terms to put the trinomial in Standard Form  $-3x^2 + 12x - 12 \rightarrow$  factor out a GCF of -3 $-3(x^2 - 4x + 4) \rightarrow$  factor the trinomial -3(x-2)(x-2) or  $-3(x-2)^2$ 72a.  $64x + x^3 - 16x^2 \rightarrow$  rearrange the terms to put the trinomial in Standard Form  $x^3 - 16x^2 + 64x \rightarrow$  factor out a GCF of x  $x(x^2 - 16x + 64) \rightarrow$  factor the trinomial x(x-8)(x-8) or  $x(x-8)^2$ 

72b.  $4xy - 4y^2 - x^2 \rightarrow$  rearrange the terms to put the trinomial in Standard Form

<b>Option 1:</b> put the $-x^2$ term first	
$-x^2 + 4xy - 4y^2 \rightarrow \text{ factor out a GCF of } -1$	-
$-(x^2 - 4xy + 4y^2) \rightarrow \text{ factor the trinomial}$	-
$-(x-2y)(x-2y)$ or $-(x-2y)^2$	-

<u>Option 2</u>: put the  $-4y^2$  term first  $-4y^2 + 4xy - x^2 \rightarrow \text{ factor out a GCF of } -1$   $-(4y^2 - 4xy + x^2) \rightarrow \text{ factor the trinomial}$  $-(2y - x)(2y - x) \text{ or } -(2y - x)^2$ 

77c.  $25x^2 - 100x + 100 \rightarrow \text{factor out a GCF of } 25$   $25(x^2 - 4x + 4) \rightarrow \text{factor the trinomial}$  $25(x - 2)(x - 2) \text{ or } 25(x - 2)^2$ 

79.

The equation  $H = -16t^2 + 64$  shows the height H of the diver t seconds after she jumps. At the exact moment she jumps, no time has passed so t = 0. To find her height at that moment, replace t with 0 and solve for H.

$$H = -16(0)^2 + 64 \rightarrow H = -16 \cdot 0 + 64 \rightarrow H = 0 + 64 \rightarrow H = 64$$
 feet

80.

When the diver hits the water, her height is 0. To find how long she has been falling when she hits the water, replace *H* with 0 and solve for *t*.

 $0 = -16t^2 + 64$ 

You will learn how to solve this type of equation in later scenarios, but for now, the only way you can solve this equation is by trying out different *t*-values and seeing if they make the equation true.

Let t = 1.  $0 = -16(1)^2 + 64 \rightarrow 0 = -16 \cdot 1 + 64 \rightarrow 0 = -16 + 64 \rightarrow 0 = 48$  $\rightarrow 0 = 48$  is a false statement so the diver does not hit the water after 1 second. Let t = 2.  $0 = -16(2)^2 + 64 \rightarrow 0 = -16 \cdot 4 + 64 \rightarrow 0 = -64 + 64 \rightarrow 0 = 0$  $\rightarrow 0 = 0$  is a true statement. The equation is true when t = 2, so the diver hits the water after falling for 2 seconds.

82a.

Since 5(0) = 0, the equation is true when x - 3 = 0. Solve the equation  $x - 3 = 0 \rightarrow x = 3$ Check:  $5(x - 3) = 0 \rightarrow 5(3 - 3) = 0 \rightarrow 5(0) = 0 \rightarrow 0 = 0 \rightarrow$ true

82b.

Since -2(0) = 0, the equation is true when x + 1 = 0. Solve the equation  $x + 1 = 0 \rightarrow x = -1$ Check:  $-2(x + 1) = 0 \rightarrow -2(-1 + 1) = 0 \rightarrow -2(0) = 0 \rightarrow 0 = 0 \rightarrow$  true

82c.

Since -3(0) = 0, the equation is true when x - 7 = 0. Solve the equation  $x - 7 = 0 \rightarrow x = 7$ Check:  $-3(x - 7) = 0 \rightarrow -3(7 - 7) = 0 \rightarrow -3(0) = 0 \rightarrow 0 = 0 \rightarrow$  true

83a. If one number is multiplied by another number and the product is 0, then one of the numbers is 0. If (x - 6)(x + 1) equals 0 then either x - 6 = 0 or x + 1 = 0. Option 1: x - 6 = 0If x - 6 = 0, then the equation is  $(0)(x + 1) = 0 \rightarrow 0$  multiplied by x + 1 is 0. Since 0 = 0, the equation is true when x - 6 = 0. If x - 6 = 0, then x = 6. Option 2: x + 1 = 0If x + 1 = 0, then the equation is (x - 6)(0) = 0. Simplify the left side: (x - 6) multiplied by 0 is 0. Since 0 = 0, the equation is true when x + 1 = 0. If x + 1 = 0, then x = -1. The equation is true for 2 different x-values: when x = 6 and when x = -1. 83b. If one number is multiplied by another number and the product is 0, then one of the numbers is 0. If (x - 10)(x - 9) equals 0 then either x - 10 = 0 or x - 9 = 0. Option 1: x - 10 = 0If x - 10 = 0, then the equation is  $(0)(x - 9) = 0 \rightarrow 0$  multiplied by x - 9 is 0. Since 0 = 0, the equation is true when x - 10 = 0. If x - 10 = 0, then x = 10. Option 2: x - 9 = 0If x - 9 = 0, then the equation is (x - 10)(0) = 0. Simplify the left side: (x - 10) multiplied by 0 is 0. Since 0 = 0, the equation is true when x - 9 = 0. If x - 9 = 0, then x = 9. The equation is true for 2 different x-values: when x = 10 and when x = 9. 83c. If one number is multiplied by another number and the product is 0, then one of the numbers is 0. If (x + 8)(x + 11) equals 0 then either x + 8 = 0 or x + 11 = 0. Option 1: x + 8 = 0If x + 8 = 0, then the equation is  $(0)(x + 11) = 0 \rightarrow 0$  multiplied by x + 11 is 0. Since 0 = 0, the equation is true when x + 8 = 0. If x + 8 = 0, then x = -8. Option 2: x + 11 = 0If x + 11 = 0, then the equation is (x + 8)(0) = 0. Simplify the left side: (x + 8) multiplied by 0 is 0. Since 0 = 0, the equation is true when x + 11 = 0. If x + 11 = 0, then x = -11. The equation is true for 2 different x-values: when x = -8 and when x = -11. 85a.  $x^2 + 7x + 10 = 0 \rightarrow$  factor the trinomial on the left side  $(x + 5)(x + 2) = 0 \rightarrow$  the left side will equal 0 if either x + 5 = 0 or x + 2 = 0. The expression x + 5 equals 0 when x = -5, and x + 2 equals 0 when x = -2. Check: x = -5 $(x+5)(x+2) = 0 \rightarrow (-5+5)(-5+2) = 0 \rightarrow (0)(-3) = 0 \rightarrow 0 = 0 \rightarrow \text{true}$ Check: x = -2 $(x+5)(x+2) = 0 \rightarrow (-2+5)(-2+2) = 0 \rightarrow (3)(0) = 0 \rightarrow 0 = 0 \rightarrow \text{true}$ The equation has 2 solutions: x = -5 and x = -2.

85b.  $x^2 - 8x + 15 = 0 \rightarrow \text{factor the trinomial on the left side}$   $(x - 5)(x - 3) = 0 \rightarrow \text{the left side will equal 0 if either } x - 5 = 0 \text{ or } x - 3 = 0.$ The expression x - 5 equals 0 when x = 5, and x - 3 equals 0 when x = 3. Check: x = 5  $(x - 5)(x - 3) = 0 \rightarrow (5 - 5)(5 - 3) = 0 \rightarrow (0)(2) = 0 \rightarrow 0 = 0 \rightarrow \text{true}$ Check: x = 3  $(x - 5)(x - 3) = 0 \rightarrow (3 - 5)(3 - 3) = 0 \rightarrow (-2)(0) = 0 \rightarrow 0 = 0 \rightarrow \text{true}$ The equation has 2 solutions: x = 5 and x = 3.

85c.

 $x^{2} + x - 20 = 0 \rightarrow \text{factor the trinomial on the left side}$   $(x + 5)(x - 4) = 0 \rightarrow \text{the left side will equal 0 if either } x + 5 = 0 \text{ or } x - 4 = 0.$ The expression x + 5 equals 0 when x = -5, and x - 4 equals 0 when x = 4. Check: x = -5  $(x + 5)(x - 4) = 0 \rightarrow (-5 + 5)(-5 - 4) = 0 \rightarrow (0)(-9) = 0 \rightarrow 0 = 0 \rightarrow \text{true}$ Check: x = 4  $(x + 5)(x - 4) = 0 \rightarrow (4 + 5)(4 - 4) = 0 \rightarrow (9)(0) = 0 \rightarrow 0 = 0 \rightarrow \text{true}$ The equation has 2 solutions: x = -5 and x = 4.

87.

One of the factors is x + 3. Solve the equation  $x + 3 = 3 \rightarrow x = 0$ The other factor is x + 4. Solve the equation  $x + 4 = 3 \rightarrow x = -1$ Check: x = 0  $(x + 3)(x + 4) = 3 \rightarrow (0 + 3)(0 + 4) = 3 \rightarrow (3)(4) = 3 \rightarrow 12 = 3 \rightarrow \text{ false}$ Check: x = -1  $(x + 3)(x + 4) = 3 \rightarrow (-1 + 3)(-1 + 4) = 3 \rightarrow (2)(3) = 3 \rightarrow 6 = 3 \rightarrow \text{ false}$ Neither x-value makes the original equation true so they are not the equation's solutions.

89.

 $(x + 3)(x + 4) = 2 \rightarrow$  the product of the factors needs to be 0 to solve the equation by setting each factor equal to 0 and solving for x.

To change the equation, start by multiplying the binomials together.

 $x^2 + 7x + 12 = 2 \rightarrow$  move the 2 to the left side by subtracting 2 on both sides

 $x^2 + 7x + 10 = 0 \rightarrow$  factor the trinomial

 $(x + 5)(x + 2) = 0 \rightarrow$  since the product of the factors is 0, one of the factors must equal 0. Set each factor equal to 0 separately and solve the equation.

Option 1: x + 5 = 0. If x + 5 = 0, then x = -5.

Option 2: x + 2 = 0. If x + 2 = 0, then x = -2.

You can check these solutions by plugging them into the original equation.

Check: 
$$x = -5$$

 $(x + 3)(x + 4) = 2 \rightarrow (-5 + 3)(-5 + 4) = 2 \rightarrow (-2)(-1) = 2 \rightarrow 2 = 2 \rightarrow \text{true}$ Check: x = -2 $(x + 3)(x + 4) = 2 \rightarrow (-2 + 3)(-2 + 4) = 2 \rightarrow (1)(2) = 2 \rightarrow 2 = 2 \rightarrow \text{true}$ 

The equation has 2 solutions: x = -5 and x = -2.

90.  $(x-2)(x+6) = 9 \rightarrow$  multiply the binomials  $x^2 + 4x - 12 = 9 \rightarrow$  move the 9 to the left side by subtracting 9 on both sides  $x^2 + 4x - 21 = 0 \rightarrow$  factor the trinomial  $(x + 7)(x - 3) = 0 \rightarrow$  since the product of the factors is 0, one of the factors must equal 0. Set each factor equal to 0 separately and solve the equation. Option 1:  $x + 7 = 0 \rightarrow \text{If } x + 7 = 0$ , then x = -7. Option 2:  $x - 3 = 0 \rightarrow If x - 3 = 0$ , then x = 3. You can check these solutions by plugging them into the original equation. Check: x = -7 $(x-2)(x+6) = 9 \rightarrow (-7-2)(-7+6) = 9 \rightarrow (-9)(-1) = 9 \rightarrow 9 = 9 \rightarrow \text{true}$ Check: x = 3 $(x-2)(x+6) = 9 \rightarrow (3-2)(3+6) = 9 \rightarrow (1)(9) = 9 \rightarrow 9 = 9 \rightarrow \text{true}$ The equation has 2 solutions: x = -7 and x = 3. 91a.  $5x + 3 = 0 \rightarrow$  subtract 3 on both sides of the equation  $5x = -3 \rightarrow$  divide by 5 on both sides  $x = -\frac{5}{5}$ 91b.  $7x - 1 = 0 \rightarrow \text{add } 1 \text{ on both sides of the equation}$  $7x = 1 \rightarrow$  divide by 7 on both sides

$$x = \frac{1}{7}$$

92a.

Since 2(0) = 0, the equation is true when 5x + 3 = 0. Solve the equation 5x + 3 = 0.  $\rightarrow x = -\frac{3}{5}$  Check:

$$2(5x+3) = 0 \rightarrow 2\left(5\left(-\frac{3}{5}\right)+3\right) = 0 \rightarrow 2(-3+3) = 0 \rightarrow 2(0) = 0 \rightarrow 0 = 0 \rightarrow \text{true}$$

92b.

Since 10(0) = 0, the equation is true when 7x - 1 = 0. Solve the equation 7x - 1 = 0.  $\rightarrow x = \frac{1}{7}$  Check:

$$10(7x - 1) = 0 \rightarrow 10\left(7\left(\frac{1}{7}\right) - 1\right) = 0 \rightarrow 10(1 - 1) = 0 \rightarrow 10(0) = 0 \rightarrow 0 = 0 \rightarrow \text{true}$$

93a.

If one number is multiplied by another number and the product is 0, then one of the numbers is 0. If (4x - 1)(x - 5) equals 0 then either 4x - 1 = 0 or x - 5 = 0. Option 1: 4x - 1 = 0If 4x - 1 = 0, then the equation is  $(0)(x - 5) = 0 \rightarrow 0$  multiplied by x - 5 is 0. Since 0 = 0, the equation is true when 4x - 1 = 0. Solve the equation  $4x - 1 = 0 \rightarrow x = \frac{1}{4}$ Option 2: x - 5 = 0If x - 5 = 0, then the equation is  $(4x - 1)(0) = 0 \rightarrow (4x - 1)$  multiplied by 0 is 0. Since 0 = 0, the equation is true when x - 5 = 0. If x - 5 = 0, then x = 5.

The equation is true for 2 different x-values: when  $x = \frac{1}{4}$  and when x = 5.

93b.

If one number is multiplied by another number and the product is 0, then one of the numbers is 0. If (x + 8)(8x - 1) equals 0 then either x + 8 = 0 or 8x - 1 = 0. Option 1: x + 8 = 0Solve the equation  $x + 8 = 0 \rightarrow x = -8$ Option 2: 8x - 1 = 0Solve the equation  $8x - 1 = 0 \rightarrow x = \frac{1}{8}$ The equation is true for 2 different x-values: when x = -8 and when  $x = \frac{1}{8}$ . 93c. There are 2 ways to make the left side of the equation equal 0. Option 1:  $3x + 2 = 0 \rightarrow$  Solve the equation.  $\rightarrow x = -\frac{2}{3}$ Option 2:  $4x - 5 = 0 \rightarrow$  Solve the equation.  $\rightarrow x = \frac{5}{4}$ 

The equation is true for 2 different x-values: when  $x = -\frac{2}{3}$  and when  $x = \frac{5}{4}$ .

94a.

 $10 = 6x^{2} + 28x \rightarrow \text{move the 10 to the right side by subtracting 10 from both sides}$   $0 = 6x^{2} + 28x - 10 \rightarrow \text{factor out a GCF of 2}$   $0 = 2(3x^{2} + 14x - 5) \rightarrow \text{factor the trinomial in the parentheses}$   $0 = 2(3x - 1)(x + 5) \rightarrow \text{The right side of the equation equals 0 if } 3x - 1 = 0 \text{ or if } x + 5 = 0.$ Option 1:  $3x - 1 = 0 \rightarrow \text{Solve the equation.} \rightarrow x = \frac{1}{3}$ Option 2:  $x + 5 = 0 \rightarrow \text{Solve the equation.} \rightarrow x = -5$ The equation has 2 solutions:  $x = \frac{1}{3}$  and x = -5.

94b.

 $20x^2 - 45 = 0 \rightarrow \text{factor out a GCF of 5}$   $5(4x^2 - 9) = 0 \rightarrow \text{factor the binomial in the parentheses}$   $5(2x + 3)(2x - 3) = 0 \rightarrow \text{The right side of the equation equals 0 if } 2x + 3 = 0 \text{ or if } 2x - 3 = 0.$ Option 1:  $2x + 3 = 0 \rightarrow \text{Solve the equation.} \rightarrow x = -\frac{3}{2}$ Option 2:  $2x - 3 = 0 \rightarrow \text{Solve the equation.} \rightarrow x = \frac{3}{2}$ The equation has 2 solutions:  $x = -\frac{3}{2}$  and  $x = \frac{3}{2}$ .

94c.  $9x^2 + 4 = 12x \rightarrow \text{move the } 12x \text{ to the left side by subtracting } 12x \text{ from both sides}$   $9x^2 - 12x + 4 = \rightarrow \text{ factor the trinomial}$   $(3x - 2)(3x - 2) = 0 \rightarrow \text{ The right side of the equation equals } 0 \text{ if } 3x - 2 = 0.$ Solve the equation 3x - 2 = 0.  $\rightarrow x = \frac{2}{3}$ Since both factors of the trinomial are (3x - 2), the equation has 1 solution:  $x = \frac{2}{3}$ .

95a.

If one number is multiplied by another number and the product is 0, then one of the numbers is 0. If x(x - 2) equals 0 then either x = 0 or x - 2 = 0. Option 1: x = 0Option 2:  $x - 2 = 0 \rightarrow$  Solve the equation.  $\rightarrow x = 2$ The equation has 2 solutions: x = 0 and x = 2.

summitmathbooks.com

95b.

If one number is multiplied by another number and the product is 0, then one of the numbers is 0. If x(x + 3) equals 0 then either x = 0 or x + 3 = 0. Option 1: x = 0Option 2:  $x + 3 = 0 \rightarrow$  Solve the equation.  $\rightarrow x = -3$ The equation has 2 solutions: x = 0 and x = -3.

95c.

If one number is multiplied by another number and the product is 0, then one of the numbers is 0. If 5x(2x + 3) equals 0 then either 5x = 0 or 2x + 3 = 0. Option 1:  $5x = 0 \rightarrow$ Solve the equation.  $\rightarrow x = 0$ Option 2:  $2x + 3 = 0 \rightarrow$ Solve the equation.  $\rightarrow x = -\frac{3}{2}$ The equation has 2 solutions: x = 0 and  $x = -\frac{3}{2}$ . 97a.  $x^2 + 2x = 0 \rightarrow$ factor out a GCF of x

 $x(x + 2) = 0 \rightarrow$  the left side will equal 0 if either x = 0 or x + 2 = 0. The expression x + 2 equals 0 when x = -2. The equation has 2 solutions: x = 0 and x = -2.

97b.

 $x^2 - 3x + 5 = 5 \rightarrow$  make the equation equal 0 by subtracting 5 on both sides  $x^2 - 3x = 0 \rightarrow$  factor out a GCF of x  $x(x - 3) = 0 \rightarrow$  the left side will equal 0 if either x = 0 or x - 3 = 0. The expression x - 3 equals 0 when x = 3. The equation has 2 solutions: x = 0 and x = 3.

97c.

 $3 = 10x^2 - 15x + 3 \rightarrow$  make the equation equal 0 by subtracting 3 on both sides  $0 = 10x^2 - 15x \rightarrow$  factor out a GCF of 5x  $5x(2x - 3) = 0 \rightarrow$  the left side will equal 0 if either 5x = 0 or 2x - 3 = 0. The expression 5x equals 0 when x = 0. The expression 2x - 3 equals 0 when  $x = \frac{3}{2}$ . The equation has 2 solutions: x = 0 and  $x = \frac{3}{2}$ .

98a.

If  $x^3(x - 11)$  equals 0 then either  $x^3 = 0$  or x - 11 = 0. Option 1:  $x^3 = 0 \rightarrow$  Solve the equation.  $\rightarrow x = \sqrt[3]{0} \rightarrow x = 0$ Option 2:  $x - 11 = 0 \rightarrow$  Solve the equation.  $\rightarrow x = 11$ The equation has 2 solutions: x = 0 and x = 11.

98b.

If  $7x^2(x + 4)$  equals 0 then either  $7x^2 = 0$  or x + 4 = 0. Option 1:  $7x^2 = 0 \rightarrow$  Solve the equation.  $\rightarrow 7x^2 = 0 \rightarrow x^2 = 0 \rightarrow x = \sqrt{0} \rightarrow x = 0$ Option 2:  $x + 4 = 0 \rightarrow$  Solve the equation.  $\rightarrow x = -4$ The equation has 2 solutions: x = 0 and x = -4.

98c. If  $2x(x^2 - 1)$  equals 0 then either 7x = 0 or  $x^2 - 1 = 0$ . Option 1:  $2x = 0 \rightarrow$  Solve the equation.  $\rightarrow 2x = 0 \rightarrow x = 0$ Option 2:  $x^2 - 1 = 0 \rightarrow$  Solve the equation.  $\rightarrow x^2 = 1 \rightarrow x = 1$  or x = -1The equation has 3 solutions: x = 0, x = 1 and x = -1100a.  $4x^2 - 21x + 5 = 0 \rightarrow$  factor the trinomial  $(4x - 1)(x - 5) = 0 \rightarrow$  The right side of the equation equals 0 if 4x - 1 = 0 or if x - 5 = 0. Option 1:  $4x - 1 = 0 \rightarrow$  Solve the equation.  $\rightarrow x = \frac{1}{4}$ Option 2:  $x - 5 = 0 \rightarrow$  Solve the equation.  $\rightarrow x = 5$ The equation has 2 solutions:  $x = \frac{1}{4}$  and x = 5. 100b.  $8x^2 + 63x - 8 = 0 \rightarrow$  factor the trinomial  $(8x - 1)(x + 8) = 0 \rightarrow$  The right side of the equation equals 0 if 8x - 1 = 0 or if x + 8 = 0. Option 1:  $8x - 1 = 0 \rightarrow$  Solve the equation.  $\rightarrow x = \frac{1}{8}$ Option 2:  $x + 8 = 0 \rightarrow$  Solve the equation.  $\rightarrow x = -8$ The equation has 2 solutions:  $x = \frac{1}{2}$  and x = -8. 100c.  $12x^2 - 7x - 10 = 0 \rightarrow$  factor the trinomial  $(4x-5)(3x+2) = 0 \rightarrow$  The right side of the equation equals 0 if 4x-5 = 0 or if 3x+2 = 0. Option 1:  $4x - 5 = 0 \rightarrow$  Solve the equation.  $\rightarrow x = \frac{5}{4}$ Option 2:  $3x + 2 = 0 \rightarrow$  Solve the equation.  $\rightarrow x = -\frac{2}{2}$ The equation has 2 solutions:  $x = \frac{5}{4}$  and  $x = -\frac{2}{3}$ . 101.  $-16t^2 + 64 = 0 \rightarrow$  factor out a GCF of -16 $-16(t^2 - 4) = 0 \rightarrow$  factor the binomial in the parentheses  $-16(t+2)(t-2) = 0 \rightarrow$  The right side of the equation equals 0 if t+2 = 0 or if t-2 = 0. Option 1:  $t + 2 = 0 \rightarrow$  Solve the equation.  $\rightarrow t = -2$ Option 2:  $t - 2 = 0 \rightarrow$  Solve the equation.  $\rightarrow t = 2$ The equation has 2 solutions: t = -2 and t = 2. 102.  $(x-6)(x-1) = 14 \rightarrow$  multiply the binomials  $x^2 - 7x + 6 = 14 \rightarrow$  move the 14 to the left side by subtracting 14 on both sides  $x^2 - 7x - 8 = 0 \rightarrow$  factor the trinomial  $(x - 8)(x + 1) = 0 \rightarrow$  Since the product of the factors is 0, one of the factors must equal 0. Set each factor equal to 0 separately and solve the equation. Option 1:  $x - 8 = 0 \rightarrow x = 8$ Option 2:  $x + 1 = 0 \rightarrow x = -1$ The equation has 2 solutions: x = 8 and x = -1.

103.

The fractions make the trinomial hard to factor. Clear the fractions by multiplying both sides by the common denominator of the 3 fractions. The common denominator of 6, 3 and 2 is 6, because 6 is the least common multiple of 2, 3 and 6.

$$\frac{1}{6}x^2 - \frac{1}{3}x - \frac{1}{2} = 0 \rightarrow \text{multiply both sides by 6}$$

$$6 \cdot \left(\frac{1}{6}x^2 - \frac{1}{3}x - \frac{1}{2}\right) = 0 \cdot 6 \rightarrow \text{distribute the 6 to all 3 terms inside parentheses}$$

$$6 \cdot \frac{1}{6}x^2 - 6 \cdot \frac{1}{3}x - 6 \cdot \frac{1}{2} = 0 \rightarrow \text{simplify each product on the left side of the equation}$$

$$x^2 - 2x - 3 = 0 \rightarrow \text{factor the trinomial}$$

$$(x - 3)(x + 1) = 0 \rightarrow \text{Since the product of the factors is 0, one of the factors must equal 0. Set each factor equal to 0 separately and solve the equation.$$

Option 1:  $x - 3 = 0 \rightarrow x = 3$ 

Option 2:  $x + 1 = 0 \rightarrow x = -1$ The equation has 2 solutions: x = 3 and x = -1.

106a.

The fractions make the trinomial hard to factor. Clear the fractions by multiplying both sides by the common denominator of the 2 fractions. The common denominator of 4 and 4 is 4.

$$\frac{1}{4}x^2 - \frac{5}{4}x + 1 = 0 \rightarrow \text{multiply both sides by 4}$$

$$4 \cdot \left(\frac{1}{4}x^2 - \frac{5}{4}x + 1\right) = 0 \cdot 4 \rightarrow \text{distribute the 4 to all 3 terms inside parentheses}$$

$$4 \cdot \frac{1}{4}x^2 - 4 \cdot \frac{5}{4}x - 4 \cdot 1 = 0 \rightarrow \text{simplify each product on the left side of the equation}$$

$$x^2 - 5x - 4 = 0 \rightarrow \text{factor the trinomial}$$

$$(x - 4)(x + 1) = 0 \rightarrow \text{Since the product of the factors is 0, one of the factors must equal 0. Set each factor equal to 0 separately and solve the equation.$$
Option 1:  $x - 4 = 0 \rightarrow x = 4$ 

Option 2:  $x + 1 = 0 \rightarrow x = -1$ The equation has 2 solutions: x = 4 and x = -1.

106b.

The trinomial is hard to factor because the third term is 8.25. If all numbers are integers, the equation will be easier to factor. 8.25 is  $8\frac{1}{4}$ , which is  $\frac{33}{4}$  when written as a mixed number.

$$x^2 - 7x + \frac{33}{4} = 0 \rightarrow \text{multiply both sides by 4}$$
  
 $4 \cdot \left(x^2 - 7x + \frac{33}{4}\right) = 0 \cdot 4 \rightarrow \text{distribute the 4 to all 3 terms inside parentheses}$   
 $4 \cdot x^2 - 4 \cdot 7x + \frac{33}{4} \cdot 4 = 0 \rightarrow \text{simplify each product on the left side of the equation}$   
 $4x^2 - 28x + 33 = 0 \rightarrow \text{factor the trinomial}$   
 $(2x - 11)(2x - 3) = 0 \rightarrow \text{Since the product of the factors is 0, one of the factors must equal 0. Set each factor equal to 0 separately and solve the equation.}
Option 1:  $2x - 11 = 0 \rightarrow x = \frac{11}{2}$   
Option 2:  $2x - 3 = 0 \rightarrow x = \frac{3}{2}$   
The equation has 2 solutions:  $x = \frac{11}{2}$  and  $x = \frac{3}{2}$ . In decimal form, the 2 solutions are 5.5 and 1.5.$ 

108.  $\frac{2x}{5} - 1 = \frac{1}{3}x - \frac{5}{6} \rightarrow \text{ multiply both sides by 30}$   $30 \cdot \left(\frac{2x}{5} - 1\right) = \left(\frac{1}{3}x - \frac{5}{6}\right) \cdot 30 \rightarrow \text{ distribute the 6 to all 3 terms inside parentheses}$   $30 \cdot \frac{2x}{5} - 30 \cdot 1 = \frac{1}{3}x \cdot 30 - \frac{5}{6} \cdot 30 \rightarrow \text{ simplify each product on both sides of the equation}$   $12x - 30 = 10x - 25 \rightarrow \text{ subtract 10x from both sides}$   $2x - 30 = -25 \rightarrow \text{ add 30 to both sides}$   $2x = 5 \rightarrow \text{ divide by 2 on both sides}$   $x = \frac{5}{2} \text{ or } 2.5$ 

109.

 $x^3 - 2x^2 = 15x \rightarrow \text{make the equation equal 0 by subtracting 15x on both sides}$   $x^3 - 2x^2 - 15x = 0 \rightarrow \text{factor out a GCF of } x$   $x(x^2 - 2x - 15) = 0 \rightarrow \text{factor the trinomial inside parentheses}$   $x(x - 5)(x + 3) = 0 \rightarrow \text{the left side will equal 0 if either } x - 5 = 0 \text{ or } x + 3 = 0$ The expression x - 5 equals 0 when x = 5 and the expression x + 3 equals 0 when x = -3. The equation has 2 solutions: x = 5 and x = -3.

110.

The Answer Key shows that the equation  $0 = -x^2 + 5x$  has two solutions: x = 0 and x = 5. There are two x-values that satisfy the equation because the rock has a height of 0 feet at two different locations: 1) at the moment it is launched and 2) when it comes back down to the ground after flying through the air. The x-value that answers the question asked is x = 5.

111.

The Answer Key shows that the equation  $1 = -16t^2 + 2t + 4$  has two solutions:  $t = -\frac{3}{8}$  and  $t = \frac{1}{2}$ . There are two *t*-values that satisfy the equation but one solution is negative so it is not a reasonable value. The positive solution represents the time the frog needs to wait before extending its tongue.

112a.

Solve the equation 2x(x - 1) = 12.  $2x(x - 1) = 12 \rightarrow \text{distribute the } 2x \text{ to both terms inside the parentheses}$   $2x^2 - 2x = 12 \rightarrow \text{move the } 12 \text{ to the left side by subtracting } 12 \text{ on both sides}$   $2x^2 - 2x - 12 = 0 \rightarrow \text{factor out a GCF of } 2$   $2(x^2 - x - 6) = 0 \rightarrow \text{factor the trinomial inside parentheses}$   $2(x - 3)(x + 2) = 0 \rightarrow \text{the left side will equal } 0 \text{ if either } x - 3 = 0 \text{ or } x + 2 = 0$ Option  $1: x - 3 = 0 \rightarrow x = 3$ Option  $2: x + 2 = 0 \rightarrow x = -2$ The equation has 2 solutions: x = 3 and x = -2.

These x-values are used to find the base and height of the rectangle. If x is negative, the base and height are negative, but a side length of a rectangle cannot have a negative measurement. The only valid solution is x = 3.

112b.

If x = 3, the base of the rectangle is 2(3), which is 6 and its height is 3 - 1, which is 2. The perimeter is the sum of all 4 sides: P = 6 + 2 + 6 + 2 = 16 inches.

113.

The length is 3 more than twice the width. If you use the variable w to represent the width, then the length is 3 more than 2w, or 2w + 3. The area of a rectangle is calculated by multiplying its length and width, so the area of this rectangle is  $(2w + 3) \cdot w$ . This can be written as w(2w + 3). Since the area of this rectangle is 44, you can make the expression w(2w + 3) equal 44 and solve the equation.  $w(2w + 3) = 44 \rightarrow$  distribute the w to both terms inside the parentheses  $2w^2 + 3w = 44 \rightarrow$  move the 44 to the left side by subtracting 44 on both sides  $2w^2 + 3w - 44 = 0 \rightarrow$  factor the trinomial  $(2w + 11)(w - 4) = 0 \rightarrow$  the left side will equal 0 if either 2w + 11 = 0 or w - 4 = 0Option 1:  $2w + 11 = 0 \rightarrow w = -\frac{11}{2}$  or -5.5Option 2:  $w - 4 = 0 \rightarrow w = 4$ The equation has 2 solutions: w = -5.5 and w = 4. These w-values represent the width of the rectangle and the width cannot be negative. The only valid solution is w = 4. Since the width is 4, the length is (2(4) + 3), which is 11. 114a.  $A = w(40 - w) \rightarrow \text{replace } w \text{ with } 10$ 

 $A = 10(40 - 10) \rightarrow A = 10(30) \rightarrow A = 300$ 

114b.

If the area is 300 ft<sup>2</sup> and the width is 10 feet, then the length is 30 feet, because  $10 \cdot 30 = 300$ . The fence surrounds the perimeter of the garden, and the total perimeter is 10 + 30 + 10 + 30 = 80 feet.

116.

The shaded region is formed by subtracting the inner rectangle from the outer rectangle. The area of the outer rectangle is (2x + 1)(2x + 3). The area of the inner rectangle is (x + 3)(x - 3). The area of the shaded region is 92 square units so you can set up the equation shown and solve for x.  $(2x + 1)(2x + 3) - (x + 3)(x - 3) = 92 \rightarrow$  multiply both pairs of binomials  $4x^2 + 8x + 3 - (x^2 - 9) = 92 \rightarrow$  distribute the subtraction to both terms inside the parentheses  $4x^2 + 8x + 3 - x^2 + 9 = 92 \rightarrow$  combine like terms  $3x^2 + 8x + 12 = 92 \rightarrow$  make the equation equal 0 by subtracting 92 on both sides  $3x^2 + 8x - 80 = 0 \rightarrow$  factor the trinomial  $(3x + 20)(x - 4) = 0 \rightarrow$  the left side will equal 0 if either 3x + 20 = 0 or x - 4 = 0Option 1:  $3x + 20 = 0 \rightarrow x = -\frac{20}{3}$ Option 2:  $x - 4 = 0 \rightarrow x = 4$ The equation has 2 solutions:  $x = -\frac{20}{3}$  and x = 4. If you replace x with  $-\frac{20}{3}$ , the inner and outer rectangles will have negative side lengths so that xvalue is not a valid solution. The only valid solution is x = 4. Since x = 4, the height of the outer rectangle is (2(4) + 3), which is 11 units. The base of the outer rectangle is (2(4) + 1), which is 9 units.

The perimeter of the outer rectangle is  $2(11) + 2(9) \rightarrow 22 + 18 \rightarrow 40$  units.

118a.  $3(x-6) = 5x - 4 \rightarrow \text{distribute the 3 to both terms inside the parentheses}$   $3x - 18 = 5x - 4 \rightarrow \text{move the "x" terms to the same side of the equation. One way to do this is to subtract 3x on both sides.$  $<math>-18 = 2x - 4 \rightarrow \text{isolate the "x" term by adding 4 on both sides}$   $-14 = 2x \rightarrow \text{divide both sides by 2}$ -7 = x

118b.

 $1 - 3(2x - 1) = 19 \rightarrow \text{distribute the } -3 \text{ to both terms inside the parentheses}$   $1 - 6x + 3 = 19 \rightarrow \text{combine the } 1 \text{ and the } +3$   $-6x + 4 = 19 \rightarrow \text{isolate the "x" term by subtracting 4 on both sides}$   $-6x = 15 \rightarrow \text{divide both sides by } -6$  $x = \frac{15}{-6} \rightarrow -\frac{15}{6} \rightarrow -\frac{5}{2} \text{ or } -2.5$ 

119a.

 $(y-7)(y+2) = 0 \rightarrow$  the left side will equal 0 if either y-7 = 0 or y+2 = 0Option 1:  $y-7 = 0 \rightarrow y = 7$ Option 2:  $y+2 = 0 \rightarrow y = -2$ The equation has 2 solutions: y = 7 and y = -2.

133a.

To find the x-intercepts, replace y with 0 and solve for x.  $x^2 + 0 = 3 \rightarrow x^2 = 3 \rightarrow x = \sqrt{3}$  or  $-\sqrt{3}$ There are 2 x-intercepts:  $(\sqrt{3}, 0)$  and  $(-\sqrt{3}, 0)$ . To find the y-intercept, replace x with 0 and solve for y.  $0^2 + y = 3 \rightarrow y = 3$ The y-intercept is (0, 3).

133b.

To find the x-intercepts, replace y with 0 and solve for x.  $0 = 4x^2 - 12x + 9 \rightarrow 0 = (2x - 3)(2x - 3) \rightarrow x = 1.5 \rightarrow$  There is one x-intercept: (1.5, 0). To find the y-intercept, replace x with 0 and solve for y.  $y = 4(0)^2 - 12(0) + 9 \rightarrow y = 9 \rightarrow$  The y-intercept is (0, 9).

135.

To find the x-intercepts, replace y with 0 and solve for x.  $0 = x^2 - 2x - 3 \rightarrow \text{ factor the trinomial}$   $0 = (x - 3)(x + 1) \rightarrow \text{ the right side equals 0 if either } x - 3 = 0 \text{ or } x + 1 = 0$ Option 1:  $x - 3 = 0 \rightarrow x = 3$ Option 2:  $x + 1 = 0 \rightarrow x = -1$ The equation has 2 solutions: x = 3 and x = -1. These are the x-values where y equals 0, so the equation has x-intercepts at (3, 0) and (-1, 0). To graph the equation, you can find other points by repeatedly plugging in an x-value and solving for the y-value that goes with it. For example, if x = 0, y = -3. Plot the point (0, -3). This is the y-

intercept. After finding the x-intercepts, you know there are points located at (3, 0) and (-1, 0). To find other points, pick easy x-values. If x = 1, y = -4. Plot the point (1, -4). If x = 2, y = -3. Plot the point (2, -3). If x = 4, y = 5. Plot the point (4, 5). If x = -2, y = 5. Plot the point (-2, 5). You can use the points as a guide to draw the curving shape of the parabola that this equation forms.

137a.

Option 1: Apply the Quotient Rule (when you divide like bases you can subtract the exponents)  $\frac{x^{-2}}{x^{-2}} \rightarrow x^{-2-(-3)} \rightarrow x^{-2+3} \rightarrow x^{1} \rightarrow x^{-2+3}$ 

$$\frac{1}{x^{-3}} \rightarrow x^{-2-(-3)} \rightarrow x^{-2+3} \rightarrow x^{-3} \rightarrow x^{-2+3}$$

Option 2: Move a term with a negative exponent to make the exponent positive. A term in the denominator moves up (to the numerator) and a term in the numerator moves down.

$$\frac{x^{-2}}{x^{-3}} \rightarrow \frac{x^3}{x^2} \rightarrow x^{3-2} \rightarrow x^1 \rightarrow x$$

Option 3: When a term looks like  $x^{-E}$ , you can rewrite it as  $\frac{1}{x^{E}}$ 

$$\frac{x^{-2}}{x^{-3}} \rightarrow \frac{\frac{1}{x^2}}{\frac{1}{x^3}} \rightarrow \frac{1}{x^2} \div \frac{1}{x^3} \rightarrow \frac{1}{x^2} \cdot \frac{x^3}{1} \rightarrow \frac{x^3}{x^2} \rightarrow x^{3-2} \rightarrow x^1 \rightarrow x$$

137b.

$$\left(-\frac{1}{3}\right)^{-3} \rightarrow \left(-\frac{3}{1}\right)^3 \rightarrow (-3)^3 \rightarrow -27$$

137c. Option 1:

$$2(x^{-1})^2 \rightarrow 2\left(\frac{1}{x}\right)^2 \rightarrow 2\cdot\frac{1}{x^2} \rightarrow \frac{2}{1}\cdot\frac{1}{x^2} \rightarrow \frac{2}{x^2}$$

Option 2:

$$2(x^{-1})^2 \rightarrow 2 \cdot x^{-2} \rightarrow 2 \cdot \frac{1}{x^2} \rightarrow \frac{2}{1} \cdot \frac{1}{x^2} \rightarrow \frac{2}{x^2}$$

138.

To find the average of 4 numbers, add them and divide by 4. Since the average of these 4 numbers is known, you can set up an equation and solve for the unknown number, *H*.

$$\frac{7+13+21+H}{4} = 10 \rightarrow \text{add the numbers in the numerator of the fraction}$$
  
$$\frac{41+H}{4} = 10 \rightarrow \text{multiply both sides by 4}$$
  
$$41 + H = 40 \rightarrow \text{subtract 41 from both sides}$$
  
$$H = -1$$

139a.

Two points on Line 1 are (0, 3) and (7, 1). To write the equation in Slope-Intercept Form, y = mx + b, you need the slope and the *y*-intercept. To find the slope, you need to know the rise and the run. The rise is the difference between the *y*-values. The run is the difference between the *x*-values. You can also refer to the slope formula below:

$$m = \frac{y_2 - y_1}{x_2 - x_1} \rightarrow m = \frac{1 - 3}{7 - 0} \rightarrow m = \frac{-2}{7} \rightarrow m = -\frac{2}{7}$$
  
The slope of the line is  $-\frac{2}{7}$ . The y-intercept is visible on the graph. It is (0, 3).  
The equation of the line is  $y = -\frac{2}{7}x + 3$ .

139b.

Two points on Line 1 are (-3, -2) and (-1, 6). To write the equation in Slope-Intercept Form, y = mx + b, you need the slope and the *y*-intercept. To find the slope, you need to know the rise and the run. The rise is the difference between the *y*-values. The run is the difference between the *x*-values. You can also refer to the slope formula below:

$$m = \frac{y_2 - y_1}{x_2 - x_1} \rightarrow m = \frac{6 - (-2)}{-1 - (-3)} \rightarrow m = \frac{8}{2} \rightarrow m = 4$$

The slope is 4, so you can write the equation as y = 4x + b. Pick one of the points on the line, replace x and y with those values and solve for b. If you use the point (-1, 6), the equation is:

6 = 4(-1) + b

You can multiply 4 and -1 to get -4, which makes the equation 6 = -4 + b, so b = 10. Now you know that the *y*-intercept is 10. In Slope-Intercept Form, the equation is y = 4x + 10.

140.

Since the average viewership is 104.3 million during the 10-year period from 2006 to 2015, you can multiply that average by 10 to find the total number of viewers over 10 years.

104.3 x 10 = 1,043 million total viewers.

Since the average viewership is 105 million during the 11-year period from 2006 to 2016, you can multiply that average by 11 to find the total number of viewers over 11 years.

105 x 11 = 1,155 million total viewers.

There were 1,043 million total viewers from 2006 to 2015 and 1,155 million total viewers from 2006 to 2016. Subtract those amounts.

1,155 – 1,043 = 112 million viewers

This difference of 112 million viewers is the number of people who watched the 2016 Super Bowl.

141a.

 $7x - 6 = \frac{1}{3}(6x - 3) \rightarrow \text{distribute the } \frac{1}{3}\text{ to both terms inside parentheses}$   $7x - 6 = 2x - 1 \rightarrow \text{subtract } 2x \text{ from both sides}$   $5x - 6 = -1 \rightarrow \text{add } 6 \text{ to both sides}$   $5x = 5 \rightarrow \text{divide by } 5 \text{ on both sides}$ x = 1

141b.

 $-x + 9 = -\frac{1}{5}x + 2 \rightarrow \text{add } x \text{ to both sides:} -\frac{1}{5}x + x \rightarrow -\frac{1}{5}x + \frac{5}{5}x \rightarrow \frac{4}{5}x$   $9 = \frac{4}{5}x + 2 \rightarrow \text{subtract } 2 \text{ on both sides}$   $7 = \frac{4}{5}x \rightarrow \text{multiply both sides by } \frac{5}{4}, \text{ since it is the reciprocal of } \frac{4}{5}$  $x = 7 \cdot \frac{5}{4} \rightarrow \frac{7}{1} \cdot \frac{5}{4} \rightarrow \frac{35}{4} \rightarrow 8\frac{3}{4} \text{ or } 8.75$ 

summitmathbooks.com