



**SUMMIT
MATH**

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ALGEBRA 1

second edition

4 OPERATIONS WITH POLYNOMIALS

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Section 3
ADDING POLYNOMIALS

GUIDED DISCOVERY SCENARIOS

9. Venture into the past for a moment. When you first learned about the operation of addition, you might have seen a classic scenario that involved adding different types of fruits.

a. If you combine 3 apples with 2 more apples, you have a total of ____ apples.

b. Adding 3 bananas to a group of 7 bananas makes a group of ____ bananas.

c. What do you have if you combine 5 apples with 5 bananas?

10. In part c. above, you cannot combine apples and bananas. They remain separate. This also applies to adding and subtracting variables. If you let a represent apples and b represent bananas, the previous statements can be written as follows:

a. $3a + 2a =$ _____

b. $3b + 7b =$ _____

c. $5a + 5b =$ _____

11. When two terms have variables and exponents, they are only like terms if the variable(s) and exponent(s) are exactly the same. Consider the expressions below.

a. $x^2 + x^2 =$ _____

b. $x^3 + 2x^3 =$ _____

c. $7x^4 + x^4 =$ _____

12. Combine like terms to simplify each expression.

a. $3x + 2x$

b. $7x^2 + 3x^2$

c. $5x^2 + 5x$

13. Simplify the following polynomials by combining like terms. State whether your final expression is a monomial, binomial, trinomial, or none of these.

a. $3x + 5y + 2y + x$

b. $7x^2 - x^2 + 3x^2$

c. $x^3 + 2x^3 + 4x^3 - 3x^2$

Section 4
***SUBTRACTING
POLYNOMIALS***

17. Consider the effect of a negative sign on the value of an expression.

- If f is -3 , what is $-1f$?
- Simplify $-1(3-7)$ in two different ways.
- If $y = 2x - 7$, then what is the value of “ $-y$ ”?

18. What is the opposite of $x + 9$?

19. Write each expression such that there are no parentheses.

a. $-(x+12)$

b. $-(-x^2+3x-2)$

c. $-(x^2+2xy-4y^2)$

In the previous scenario, the negative sign ($-$) to the left of each expression in parentheses indicates that you multiply by -1 . The -1 is treated like a coefficient and when a coefficient is -1 , as in “ $-1x$ ”, you only need to write the ($-$) sign. Mathematicians like to save time, when possible.

20. If $-(x-2)$ can be written as $-x + 2$, then what is another way to write $3-(x-2)$?

21. Write an expression that is equivalent to $x+3-(x-y)$ and has no parentheses.

22. Subtract the following expressions.

a. $(x+11)-(x+3)$

b. $(5y^2-20)-(2y^2-5)$

c. $x^2-(5x-1)$

Section 5
***MULTIPLYING
POLYNOMIALS: PART 1***

GUIDED DISCOVERY SCENARIOS

29. In a previous scenario, $(2x-5y)-(2x+5y)$, you distributed the negative sign through the second set of parentheses to remove the parentheses. Multiplying a polynomial by -1 , or by any number, for that matter, requires the use of the Distributive Property. As a matter of review, use the Distributive Property to simplify each expression shown.

a. $3(x-7)$

b. $2(3x+4)$

c. $-9(3x^2-7x+5)$

30. Fill in the blanks below.

a. $5x \cdot 5$ is the same as $5 \cdot x \cdot 5$ which can be rearranged as $5 \cdot 5 \cdot x$ using the _____ Property. (If you don't know, just check the Answer Key.)

b. $5x \cdot 6x$ is the same as $5 \cdot x \cdot 6 \cdot x$ which can be rearranged as _____ using the _____ Property.

c. $5x \cdot 7x^2$ is the same as _____ which can be rearranged as _____ using the _____ Property.

d. Simplify the expression $5x^2 \cdot 3x^3$.

31. Simplify the following multiplication scenarios.

a. $3x \cdot 2x^2$

b. $3x \cdot -4x$

c. $3x \cdot 6$

32. Use the Distributive Property to simplify the following expression.

$$3x(2x^2 - 4x + 6)$$

33. Multiply the expressions below.

a. $-2x^2 \cdot 3x^3$

b. $-2x^2 \cdot -4x^2$

c. $-2x^2 \cdot 7x$

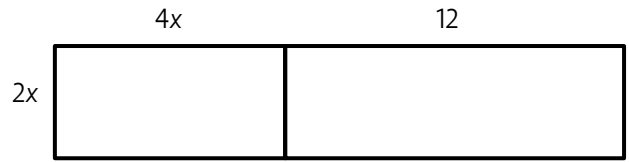
d. $-2x^2 \cdot -2$

34. Use the Distributive Property to simplify the following expression.

$$-2x^2(3x^3 - 4x^2 + 7x - 2)$$

GUIDED DISCOVERY SCENARIOS

35. Let's consider the Distributive Property from another perspective. Determine the area of each rectangle shown.

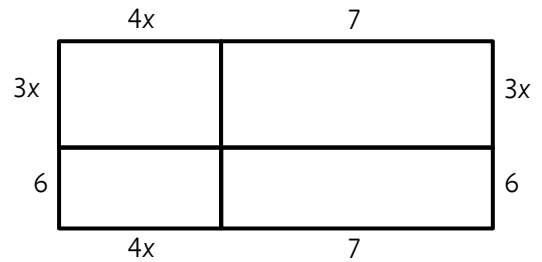


36. The rectangle shown is the same size as the entire figure in the previous scenario.



- How long is the base?
- What is the area of this rectangle?

37. Determine the area of each rectangle shown and write each area inside its rectangle.



38. The rectangle shown is the same size as the entire figure in the previous scenario.



- What expression represents the length of the base?
- What expression represents the height?
- What is the area of this rectangle?

39. The previous scenarios illustrate that the Distributive Property can be applied to the area of a rectangle. When the rectangle is split into four separate regions, the total area is $4x \cdot 3x + 4x \cdot 6 + 7 \cdot 3x + 7 \cdot 6$, which simplifies to _____.

In the previous scenario where the rectangle is not broken apart, it has a base of $4x + 7$ and a height of $3x + 6$ so its area is $(4x + 7)(3x + 6)$. The second rectangle is the same size as the first rectangle so these two rectangles have equal areas. It follows that $(4x + 7)(3x + 6) = 4x \cdot 3x + 4x \cdot 6 + 7 \cdot 3x + 7 \cdot 6 = 12x^2 + 45x + 42$

Section 8
***WHEN THE PRODUCT OF
BINOMIALS IS A BINOMIAL***

62. Multiply the following binomials.

a. $(x-1)(x+1)$

b. $(x-2)(x+2)$

c. $(x+3)(x-3)$

63. Multiply the following binomials.

a. $(x-4)(x+4)$

b. $(x+5)(x-5)$

c. $(x-6)(x+6)$

64. Multiply the following binomials.

a. $(x-y)(x+y)$

b. $(2x+3y)(2x-3y)$

65. Multiply the following binomials.

a. $(6x^2-7)(6x^2+7)$

b. $\left(\frac{1}{2}x+\frac{1}{3}\right)\left(\frac{1}{2}x-\frac{1}{3}\right)$

66. In the previous two groups of multiplication scenarios, how many terms were in your final answer? Write out an explanation that explains why your final answer contains this many terms.

67. Use the pattern that you discovered to quickly multiply the following binomials.

a. $(5x-1)(5x+1)$

b. $(4x+y)(4x-y)$

c. $\left(x-\frac{2}{7}\right)\left(x+\frac{2}{7}\right)$

68. Multiply the following expressions.

a. $(A+B)(A-B)$

b. $(x+y)(x-y)$

Section 12

ANSWER KEY

1.	a. $A = 30$ b. $A = 20$ c. $m = -5$
2.	a. 2.8 inches b. higher after 5 inches (height of 6 in. versus a height of 5.8 in.)
3.	Circle $3x$ and $\frac{x}{3}$
4.	Circle $3xy$
5.	Circle $f + g$ and $2f + 3g$
6.	Circle $\frac{f+g}{3}$ and $fg + 5xy$
7.	Circle $x + y + z$ and $x^2 - 5x + 7$
8.	a. 15 feet b. 16 feet c. The height would be negative so the ball is on the ground d. 7 feet
9.	a. 5 b. 10 c. 5 apples and 5 bananas
10.	a. $5a$ b. $10b$ c. $5a + 5b$
11.	a. $2x^2$ b. $3x^3$ c. $8x^4$
12.	a. $5x$ b. $10x^2$ c. $5x^2 + 5x$
13.	a. $4x + 7y$; binomial b. $9x^2$; monomial c. $7x^3 - 3x^2$; binomial
14.	a. $4x^2 + 3x - 3$; trinomial b. $4x^2y + 7xy^2$; binomial
15.	a. $6x^2$; monomial b. $4x^2 - 8x + 4$; trinomial
16.	a. $x^3 + 10x^2 - 5$; trinomial b. $3x^3 - 3x^2 - 4x - 10$; none of these
17.	a. 3 b. $-1(-4) = 4$ or $-3 + 7 = 4$ c. $-2x + 7$
18.	$-x - 9$
19.	a. $-x - 12$ b. $x^2 - 3x + 2$ c. $-x^2 - 2xy + 4y^2$
20.	$3 - x + 2 \rightarrow -x + 5$ or $5 - x$
21.	$x + 3 - x + y$ or $3 + y$
22.	a. $x + 11 - x - 3 \rightarrow 8$ b. $5y^2 - 20 - 2y^2 + 5 \rightarrow 3y^2 - 15$ c. $x^2 - 5x + 1$
23.	a. $-10y$, monomial b. $-3x^2 + y^2$, binomial c. $-2x^2 + 5$, binomial
24.	a. $-3x^3 + 8x^2 - 6x - 5$, none b. $-x^2 - xy + 2y^2$, trinomial

25.	$-2x^2 + 2.1x - 4.5$
26.	$24 - x^2$
27.	$2 + 4x - 3x^2 + 7 \rightarrow -3x^2 + 4x + 9$
28.	a. $5x + 30$ b. $-30x + 70$
29.	a. $3x - 21$ b. $6x + 8$ c. $-27x^2 + 63x - 45$
30.	a. Commutative b. $5 \cdot 6 \cdot x \cdot x$; Commutative c. $5 \cdot x \cdot 7 \cdot x^2$; $5 \cdot 7 \cdot x \cdot x^2$; Commutative d. $15x^5$
31.	a. $6x^3$ b. $-12x^2$ c. $18x$
32.	$6x^3 - 12x^2 + 18x$
33.	a. $-6x^5$ b. $8x^4$ c. $-14x^3$ d. $4x^2$
34.	$-6x^5 + 8x^4 - 14x^3 + 4x^2$
35.	$8x^2$ $24x$
36.	Base: $4x + 12$; Area: $8x^2 + 24x$
37.	$12x^2$ $21x$ $24x$ 42
38.	a. $4x + 7$ b. $3x + 6$ c. $12x^2 + 45x + 42$
39.	$12x^2 + 45x + 42$
40.	$6x^3$ $18x^2$ $9x$ $4x^2$ $12x$ 6
41.	Area = $6x^3 + 22x^2 + 21x + 6$
42.	lowest
43.	$-2x^4$ $-x^3$ $3x^2$ $7x$ 1
44.	a. $x^3 + x^2 - x$ b. $5y^2 + 3y - 4$
45.	a. $5x^4 - 3x^2 + 2$ b. $2x^2 + 3x - 20$
46.	Think about exponents. 7 can be written as $7x^0$, since x^0 is 1. Thus, 7 has the lowest exponent.
47.	a. $3x^2 + 4x - 1$ b. $-7x^4 + 4x^3 + 2x + 6$ c. $-x^3 - 5x^2 - 2x + 5$
48.	$4x^3 + x^2y - 2xy^2 + 3y^3$ or $3y^3 - 2xy^2 + x^2y + 4x^3$
49.	a. $6x - 14y$