SUMMIT MATH

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ALGEBRA 1

second edition

5

FACTORING POLYNOMIALS & SOLVING QUADRATIC EQUATIONS

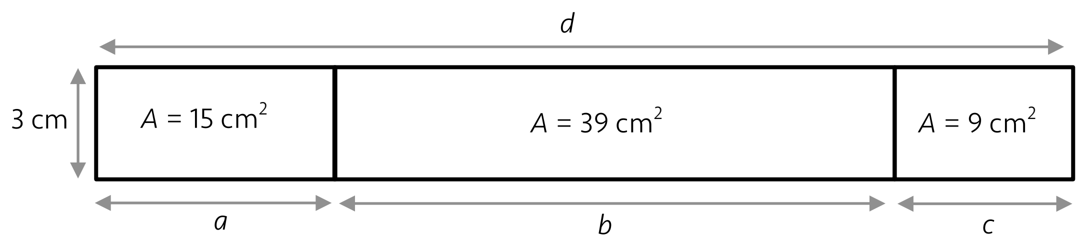
CONTENTS

Section 1

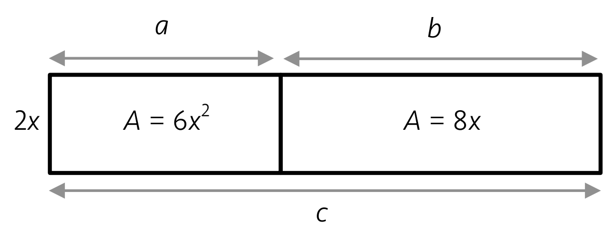
REVIEW MULTIPLYING POLYNOMIALS

In later scenarios, you will eventually learn about a topic known as factoring. However, you first need to review how multiplying polynomials forms new polynomials.

1. Given the areas of the rectangles below, determine the values of a, b, c and d.



2. The area of the entire rectangle above can be expressed as 63 cm^2. If the area is written as the base times the height, it can be expressed as $(21 \cdot 3)$ cm^2. We'll be looking more closely at this later. Given the areas of the rectangles below, determine the lengths of a, b, and c.



The area of the entire rectangle above can be expressed as $6x^2+8x$. If the area is written as the base times the height, it can be expressed as $(3x+4)(2x)$ or $2x(3x+4)$. The base is a binomial, $3x + 4$, and the height is a monomial, $2x$, to review some vocabulary from earlier lessons.

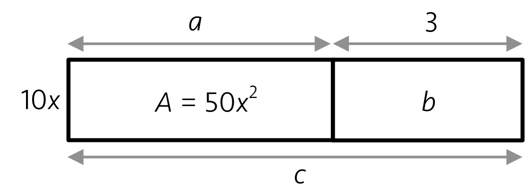
3. Multiply each expression.

a. $x(x+2)$ b. $3x(7x-5)$ c. $-4x(2x^2-x-6)$

4. Rewrite each expression as the product of a monomial and a binomial. How can you be sure that your product is a correctly rewritten form of the original expression?

a. $5x^2-20x$ b. $6y^2+15y$

5. What are the values of a, b, and c, in the rectangle shown? Two of the values are side lengths, and the other value is the area of the smaller rectangle.



6. Draw a rectangle that has the following area. Label the length of the base and height.

a. $11x+33$

b. y^2+y

7. Rewrite each of the areas in the previous scenario as the product of a monomial and a binomial.

A brief lesson on vocabulary: when you rewrite an expression as the product of two other expressions, as in the previous scenario, the two expressions are called factors. In this context, the word *factor* is a noun. Additionally, the process of rewriting an expression as the product of its factors is often referred to as factoring. In another context, then, the word *factor* is a verb.

8. Write each polynomial as the product of two factors.

a. $7x+7$

b. $10x^2-35x+5$

9. Factor each polynomial.

a. $3x^2-6x-5$

b. $8x^3+16x^2+12x$

Identifying a common factor can lead to different opinions. Consider the polynomial $8x^3+20x^2+12x$. The three terms of this trinomial share 2 as a common factor, as well as 4 and $4x$. In situations like this, it is typical to identify $4x$ as the greatest common factor (GCF).

10. The terms in the expression below have many factors in common. Identify all of the common factors of the terms in each expression.

a. $15x^3+30x^2+45x$

b. $90x^3-30x^2+20x^4$

Section 2

WRITING A TRINOMIAL AS A PRODUCT OF TWO BINOMIALS

17. In earlier lessons, you learned how to multiply binomials. Bring your mind back to that topic by multiplying the following binomials. Notice how the result relates to the original binomials.

a. $(x+2)(x-7)$ b. $(x-6)(x+6)$ c. $(x-3)^2$

18. Look at your work for the 3 multiplication scenarios in the previous scenario. Could you do that work in reverse? If you started with your final expression, could you work backwards to figure out the two binomials that you multiplied to make that expression? Try to do that with the following expressions. Write each expression as the product of two binomials.

a. x^2-2x-3 b. x^2-25 ★c. $x^2-8x+16$

19. What value of A makes the two expressions equivalent?

a. $x^2+Ax-18$ and $(x+6)(x-3)$

b. $x^2+Ax-121$ and $(x+11)(x-11)$

c. $x^2+Axy+9y^2$ and $(x-3y)^2$

20. It may be helpful to go through the mental cycle of multiplying binomials again. As before, multiply the following binomials. Try to make connections between your final expression and the original binomials that you multiplied.

a. $(x+3)(x-5)$ b. $(x-7)(x+7)$ c. $(x+2y)^2$

21. Once again, try to work backwards to write each expression as the product of two binomials.

a. $x^2+7x+10$ b. x^2-y^2 ★c. $y^2-12y+36$

Section 5

USING FACTORING TO SOLVE EQUATIONS

Let's take a break from factoring to consider some scenarios that contain polynomials like the ones we have been working with so far.

69. Suppose you attempt a basketball foul shot. The path of the ball as it travels through the air is modeled by the equation $H=-16t^2+20t+6$, where H is the height of the ball, measured in feet, t seconds after it leaves your hand. What is the height of the ball at the exact moment the ball is released?

70. In the previous scenario, suppose the shot misses the basket completely and hits the ground below the basket. How long after the ball is released does it hit the ground? Try to set up an equation that would help you answer this question. If this is confusing, keep reading.

When the ball hits the ground, its height is 0. In the equation $H=-16t^2+20t+6$, if you replace H with 0, the equation becomes $0=-16t^2+20t+6$. You can try to isolate "t" in this equation, but you probably do not know how to solve this type of equation yet.

In the next scenarios, you will learn about equations that involve w^2, t^2, n^2, or even x^2. First, consider this: When you solve an equation like $2x - 6 = 17$, the variable appears in only one term. As a result, you can undo the operations to isolate the variable x. You can add 6 to undo the subtracted 6 and you can divide by 2 to undo the multiplied 2. Once you understand how to undo every operation, you can <u>always</u> isolate x in this type of equation.

71. Now consider an equation like $x^2+5x=-6$.

   a. You can try to combine the two terms containing "x" but they are not **like** terms.

   b. If you undo the multiplied 5 in "$5x$" by dividing both sides by "5" you will end up with $\frac{x^2}{5}+x=\frac{-6}{5}$. This looks even more confusing.

   c. You can factor out an x to get $x(x+5)=-6$ and then divide both sides by $(x + 5)$, but the equation becomes $x=\frac{-6}{x+5}$, which also looks very confusing.

You can keep trying to isolate the variable to solve the equation, but you need a new strategy.

72. Start by moving all of the terms to the left side of the equation to make it look like $x^2+5x+6=0$. Now put your factoring practice to work and rewrite the trinomial as the product of two factors.

73. When the trinomial is factored, the equation becomes $(x+3)(x+2)=0$. As a reminder, the goal is to find all possible values of *x* that make the equation true. Spend some time trying out different numbers until you find a number that makes the left side of the equation equal 0.

74. If it is difficult for you to find *x*-values that make the previous equation equal 0, it is easier if you think about each factor separately. The expression $(x+3)(x+2)$ has a value of "0" if $x+3=0$ or if $x+2=0$.

a. $x+3$ equals 0 if *x* = ______.

b. $x+2$ equals 0 if *x* = ______.

75. In the previous scenario, *x* can equal 2 values: −2 or −3. This means that the original equation, $x^2+5x=-6$, has 2 solutions. Check both of these solutions to confirm they are solutions.

a. Replace *x* with −2.

$(\quad)^2+5(\quad)=-6$

b. Let *x* = −3.

$(\quad)^2+5(\quad)=-6$

76. When you first learn how to solve equations, you typically see that an equation has 1 solution. In this book, you learn that some equations have 2 solutions (or more...). Looking at each equation below, how many solutions does the equation have? Do not solve each equation.

a. $x-7=0$

b. $(x+5)(x-10)=0$

c. $(x-5)(x+3)(x-2)=0$

77. When a polynomial is written in its factored form, it is easier to figure out how to make the polynomial equal 0, because you can look at each factor separately. Fill in the blanks below.

a. The expression $(x-3)(x-4)$ has a value of "0" if _________ $=0$ or if _________ $=0$.

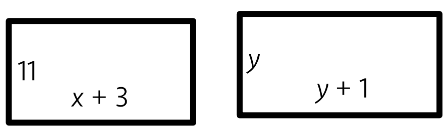
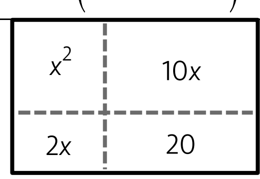
b. The expression $(x+2)(x+5)$ is equal to "0" if _________ $=0$ or if _________ $=0$.

c. The equation $(3x-1)(2x+3)=0$ is true if _________ $=0$ or if _________ $=0$.

78. Solve this puzzle: There are two numbers. When you multiply the numbers together, their product is zero. One of the numbers is 7. What is the other number?

Section 10

ANSWER KEY

1.	a. 5 cm b. 13 cm c. 3 cm d. 21 cm
2.	a. $3x$ b. 4 c. $(3x + 4)$
3.	a. x^2+2x b. $21x^2-15x$ c. $-8x^3+4x^2+24x$
4.	a. $5x(x-4)$ b. $3y(2y+5)$
5.	$a = 5x$, $b = 30x$, $c = 5x + 3$
6.	
7.	a. $11(x+3)$ b. $y(y+1)$
8.	a. $7(x+1)$ b. $5(2x^2-7x+1)$
9.	a. Prime (not factorable) b. $4x(2x^2+4x+3)$
10.	a. 3, 5, 15, x, $3x$, $5x$, $15x$ b. 2,5,10, x,$2x$,$5x$,$10x$, x^2,$2x^2$,$5x^2$,$10x^2$
11.	a. $15x$ b. $10x^2$
12.	a. $2x(x+1)$ b. $6x(2x^2+5x-4)$ c. $8x^2(2x^2-10x-5)$
13.	$6x^2(3x^2+10x-5)$
14.	
15.	a. $4x$ b. $x^2 + 10x + 24$
16.	a. $x^2 + 12x + 27$ b. $(x+9)(x+3)$
17.	a. $x^2-5x-14$ b. x^2-36 c. x^2-6x+9
18.	a. $(x+1)(x-3)$ b. $(x+5)(x-5)$ c. $(x-4)^2$
19.	a. 3 b. 0 c. −6
20.	a. $x^2-2x-15$ b. x^2-49 c. $x^2+4xy+4y^2$
21.	a. $(x+2)(x+5)$ b. $(x+y)(x-y)$ c. $(y-6)^2$

22.	a. 4 b. −5 c. −12
23.	$4(x^2+4x+3) \rightarrow 4x^2+16x+12$
24.	When you distribute the 4 to both binomials, the result is 4 times larger than it should be.
25.	$-2(x^2-4x-5) \rightarrow -2x^2+8x+10$
26.	a. −2 b. −3 c. −3
27.	Neither; $2(x-6)(x+1)$ is correct.
28.	a. $(x+2)(x+5)$ b. $(x+7)(x+1)$ c. $(x+4)(x+31)$
29.	a. $(x-2)(x+9)$ b. $(x-6)(x+1)$ c. $(x-5)(x+32)$
30.	a. $(x+2)(x+1)$ b. $(x-2)(x-1)$ c. $(x-2)(x+1)$ d. $(x+2)(x-1)$
31.	a. $4x(x-5)$ b. $-3(2x^2-6x+1)$ c. $7x(2x^2-x-5)$
32.	a. $(x-12)(x-1)$ b. $(x-3)(x-8)$ c. $(x-40)(x-15)$
33.	a. $(x+7)(x-1)$ b. $100(x+7)(x-1)$
34.	a. $x^3(x+7)(x-1)$ b. $-2(x+7)(x-1)$ c. $-3(x+7)(x-1)$
35.	a. $2(x+3)(x-1)$ b. $-3(x+5)(x+2)$
36.	$(3x+7)(x-1)$
37.	a. 3; $3x^2-x+1$ b. 1; x^2+2x-9 c. −1; $-x^2-8x+3$
38.	a. $(2x+1)(x+5)$ b. $(x+5)(3x+1)$ c. $(2x+5)(2x-3)$
39.	a. −22 b. 8
40.	a. $(3x-2)(x+3)$ b. $(2x-7)(2x-1)$ c. $-(5x-3)(x+4)$ or $(-5x+3)(x+4)$
41.	a. $(2x+3)(x-2)$ b. $10(2x+3)(x-2)$

42.	a. $x(2x+3)(x-2)$ b. $-3(2x+3)(x-2)$ c. $-5(2x+3)(x-2)$
43.	The last term would need to be "+6".
44.	If you factor out the GCF first, you end up with $2(x-2)(x+10)$.
45.	a. $(x+2)(x-2)$ b. $(x+3)(x-3)$ c. $(4+x)(4-x)$
46.	a. $(5+x)(5-x)$ b. $(x+7)(x-7)$ c. $(9+x)(9-x)$
47.	a. $(x+10)(x-10)$ b. $(y+5)(y-5)$ c. $(m+20)(m-20)$
48.	a. $2(x+2)(x-2)$ b. $2(x+3)(x-3)$ c. $3(4+x)(4-x)$
49.	a. $3(5+x)(5-x)$ b. $x(x+7)(x-7)$ c. $-2(x-6)(x+6)$
50.	It cannot be factored. It is prime.
51.	a. $(2x+7)(2x-7)$ b. $(4x+5y)(4x-5y)$
52.	a. $(3f+8)(3f-8)$ b. $4(4g+1)(4g-1)$ c. It cannot be factored. It is prime.
53.	a. $(x^2+9)(x+3)(x-3)$ b. $(x^2+1)(x+1)(x-1)$
54.	$(x^4+1)(x^2+1)(x+1)(x-1)$
55.	a. $(x+1)^2$ b. $(x-2)^2$ c. $(x-3)^2$
56.	a. $(2x+5)^2$ b. $(1-5x)^2$ or $(5x-1)^2$ c. $9(2-x)^2$ or $9(x-2)^2$
57.	The factored form of each trinomial contains a binomial that is squared.
58.	a. $(y-x)^2$ b. $(2x-7y)^2$
59.	a. $2(x-7)^2$ b. $-x(x+9)^2$ c. $-(x-5y)^2$
60.	$A^2 + 2AB + B^2$
61.	$A^2 - 2AB + B^2$
62.	$(A-10B)^2$
63.	$x \cdot x$ is not $2x^2$
64.	If you factor out the GCF first, it becomes $x(x-9)(x+2)$.
65.	Both are correct
66.	a. $(A+B)^2$ b. $(A-B)^2$ c. $(3x-5)^2$
67.	In a perfect square trinomial, the middle term is twice the product of the square roots of the first and third terms. Stated another way, find the product of the first and third terms. Take the square root of that result. If the middle term is twice the value of that square root, the trinomial is a perfect square trinomial.
68.	a. $(2x-11)(x+1)$ b. $(3x-2)(x-4)$ c. $(2x-1)(2x+7)$
69.	6 ft; In the equation, replace t with 0.
70.	You could solve the equation $0=-16t^2+20t+6$, but you do not know how to do this yet. This is intentional. In the coming scenarios, you will learn how to solve this type of equation.
71.	–
72.	$(x+3)(x+2)$
73.	$x = -3$ or $x = -2$
74.	a. $x = -3$ b. $x = -2$
75.	a. $(-2)^2+5(-2)=-6 \rightarrow 4-10=-6 \rightarrow$ TRUE b. $(-3)^2+5(-3)=-6 \rightarrow 9-15=-6 \rightarrow$ TRUE
76.	a. 1 solution b. 2 factors: 2 solutions c. 3 factors: 3 solutions
77.	a. $x-3=0$; $x-4=0$ b. $x+2=0$; $x+5=0$ c. $3x-1=0$; $2x+3=0$
78.	0
79.	a. 1 b. −1 c. 7
80.	Each equation has 2 solutions. a. x = 2 or −3 b. x = −7 or 1 c. x = 4 or 5
81.	There are infinitely many options. $A=1, B=1$ or $A=2, B=\frac{1}{2}$ or $A=3, B=\frac{1}{3}$ or $A=4, B=\frac{1}{4}$ etc...
82.	x = 3 or −2 but these solutions do not make the original equation true.
83.	#2; The product equals "0," which forces one of the factors to be "0."
84.	a. x = −3 or −1 b. x = −6 or −4 c. x = 8 or −2
85.	Factor each trinomial. When factored, the equations in this scenario become the equations in the previous scenario. a. x = −3 or −1 b. x = −6 or −4 c. x = 8 or −2
86.	$(x-5)(x-2)=0 \rightarrow x$ = 5 or 2
87.	Rewrite the equation to make it equal 0.