SUMMIT MATH

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ALGEBRA 1

second edition

7 RADICAL EXPRESSIONS & EQUATIONS

CONTENTS

Section 1

EXPONENTS AND ROOTS

In this book, you will learn about radical expressions like the square root of 49, or $\sqrt{49}$. To understand square roots, you need to be familiar with exponents, which represent repeated multiplication. Let's go back to that topic to prepare your mind to learn about radical expressions.

1. Simplify the following expressions by raising each base to an exponent.

   a. 7^2 b. 5^3 c. 4^4 d. 2^5

2. When you learn about radical expressions, you learn how to think about exponents (or repeated multiplication) in reverse. Consider the following questions.

   a. What number when squared (raised to the 2nd power) is equal to 49?

   b. What number when cubed (raised to the 3rd power) is equal to 125?

   c. What number when raised to the 4th power is equal to 256?

   d. What number when raised to the 5th power is equal to 32?

The calculations you did in scenario 1 should have made it easy to answer the questions in scenario 2. In fact, the calculations in scenario 1 *must* be done in order to answer the questions in scenario 2.

3. At this point, we need to define some vocabulary.

   a. Since 7^2 (or 7 squared) is 49, 7 is called the _____ of 49.

   b. Since 5^3 (or 5 cubed) is 125, 5 is called the _____ of 125.

   c. Since 4^4 (or 4 to the fourth power) is 256, 4 is called the _____ of 256.

   d. Since 2^5 (or 2 to the fifth power) is 32, 2 is called the _____ of 32.

   e. If $A^n = B$, then A is the _____ of B.

4. In mathematics, when there are many ways to show a concept, the most concise form will likely be used more often. In the table below, there are 3 equivalent (equal) forms of the same statement.

The 2nd Form is how you can interpret the question stated in the 1st Form.

The 3rd Form is the more concise version of the 1st Form.

1st Form	2nd Form	3rd Form
What is the square root of 49?	What number when squared is 49?	Simplify $\sqrt{49}$.
What is the cube root of 125?	What number when cubed is ____?	Simplify $\sqrt[3]{\quad}$.
What is the fourth root of 256?	What number raised to the 4th power is ____?	Simplify $\sqrt[4]{\quad}$.
What is the fifth root of 32?	What number raised to the 5th power is ____?	Simplify $\sqrt[5]{\quad}$.
What is the *n*th root of *B*?	What number raised to the *n*th power is ____?	Simplify $\sqrt[n]{\quad}$.

5. Rewrite the following statements in a more concise form (the 3rd Form).

a. What is the square root of 81?

b. What is the cube root of 64?

c. What is the fourth root of 81?

d. What is the fifth root of 1?

6. Simplify the following expressions by raising each base to its specified exponent.

a. 2^3 b. 3^3 c. 4^3 d. 5^3

7. Use your work in the previous scenario to simplify the following expressions.

a. $\sqrt[3]{8}$ b. $\sqrt[3]{27}$ c. $\sqrt[3]{64}$ d. $\sqrt[3]{125}$

8. Simplify the following expressions by raising each base to its specified exponent.

a. 2^4 b. 3^4 c. 4^4 d. 5^4

9. Use your work in the previous scenario to simplify the following expressions.

a. $\sqrt[4]{16}$ b. $\sqrt[4]{81}$ c. $\sqrt[4]{256}$ d. $\sqrt[4]{625}$

Section 3

SIMPLIFYING RADICAL EXPRESSIONS

The next scenarios use your familiarity with rational square roots to help you learn how to simplify square roots that are irrational.

31. Simplify each product as much as possible. Analyze the results as you work through each column.

a. $\sqrt{4\cdot 9}$	c. $\sqrt{25\cdot 4}$	e. $\sqrt{16\cdot 25}$	g. $\sqrt{16\cdot 9}$
b. $\sqrt{4}\cdot\sqrt{9}$	d. $\sqrt{25}\cdot\sqrt{4}$	f. $\sqrt{16}\cdot\sqrt{25}$	h. $\sqrt{16}\cdot\sqrt{9}$

32. Simplify each product as much as possible. Analyze the results as you work through each column.

a. $\sqrt{16\cdot 4}$	c. $\sqrt{100\cdot 49}$	e. $\sqrt{36\cdot 64}$	g. $\sqrt{81\cdot 25}$
b. $\sqrt{16}\cdot\sqrt{4}$	d. $\sqrt{100}\cdot\sqrt{49}$	f. $\sqrt{36}\cdot\sqrt{64}$	h. $\sqrt{81}\cdot\sqrt{25}$

33. Your work in the previous scenario should suggest the following properties:

a. $\sqrt{A\cdot B}$ is equivalent to ... b. $\sqrt{X}\cdot\sqrt{Y}$ is equivalent to ....

34. Now consider a product such as $\sqrt{25}\cdot\sqrt{3}$. You can simplify $\sqrt{25}$ as 5, but you cannot rewrite $\sqrt{3}$. As a result, the most simplified form of $\sqrt{25}\cdot\sqrt{3}$ is _____.

35. What is the most simplified form of $\sqrt{4}\cdot\sqrt{5}$?

Section 6

ADDITION AND SUBTRACTION WITH RADICALS

83. Simplify each sum or difference as much as possible.

a. $\sqrt{1+4}$

b. $\sqrt{1}+\sqrt{4}$

c. $\sqrt{9-1}$

d. $\sqrt{9}-\sqrt{1}$

e. $\sqrt{4+12}$

f. $\sqrt{4}+\sqrt{12}$

84. Is the expression $\sqrt{1}+\sqrt{1}$ equal to the expression $\sqrt{1+1}$?

85. Simplify each sum or difference as much as possible.

a. $\sqrt{9+16}$

b. $\sqrt{9}+\sqrt{16}$

c. $\sqrt{100-25}$

d. $\sqrt{100}-\sqrt{25}$

e. $\sqrt{100-64}$

f. $\sqrt{100}-\sqrt{64}$

86. Determine if the following properties exist.

a. $\sqrt{A}+\sqrt{B}=\sqrt{A+B}$

b. $\sqrt{A}-\sqrt{B}=\sqrt{A-B}$

87. At this point you have seen that $\sqrt{2}\cdot\sqrt{3}$ is equivalent to $\sqrt{2\cdot3}$, but your work above reveals that $\sqrt{2}+\sqrt{3}$ does not equal $\sqrt{2+3}$. In mathematics, there are many instances where terms can be multiplied together, but they cannot also be added together.

a. For example, $3x+2x$ is $5x$, but $3x+2y$ cannot be combined. What is the simplified form of $3x\cdot2y$?

b. The expression $y+3y+7y$ can be simplified, but $4M+2N$ cannot be combined. What is the simplified form of $4M\cdot2N$?

c. The expression $xy^4+3xy^4+5xy^4$ can be simplified, but $5xy^2+6x^2y$ cannot. What is the simplified form of $5xy^2\cdot6x^2y$?

88. Explain why each statement below is false.

a. $\sqrt{4}+\sqrt{4}=\sqrt{8}$ b. $\sqrt{4}+\sqrt{9}=\sqrt{13}$ c. $\sqrt{36}-\sqrt{25}=\sqrt{11}$

89. Simplify each expression as much as you can.

a. 2 eggs plus 3 eggs b. $2x+3x$ c. $2x^2+3x^2$ d. $2\sqrt{2}+3\sqrt{2}$

90. Simplify each expression as much as you can.

a. $1\sqrt{2}+1\sqrt{2}$ b. $\sqrt{3}+\sqrt{3}$ c. $\sqrt{5}+2\sqrt{5}$

91. Simplify each expression as much as you can.

a. $6\sqrt{2}-4\sqrt{2}$ b. $10\sqrt{3}-7\sqrt{3}$ c. $2\sqrt{5}-6\sqrt{5}$

92. What is the simplified form of $\sqrt{5}+\sqrt{6}$?

93. Explain how you can determine if two radical expressions are like terms.

94. The expressions $\sqrt{20}$ and $\sqrt{45}$ cannot be combined in their current forms. However, if you simplify each radical expression as much as possible, it becomes clear that they can be combined. In a different form, the numbers become like terms. What is the simplified form of $\sqrt{20}+\sqrt{45}$?

Section 13
ANSWER KEY

1.	a. 49 b. 125 c. 256 d. 32
2.	a. 7 b. 5 c. 4 d. 2
3.	a. square root b. cube root c. 4th root d. 5th root e. *n*th root
4.	Row 1: no blanks need to be filled Row 2: fill in both blanks with 125 Row 3: fill in both blanks with 256 Row 4: fill in both blanks with 32 Row 5: fill in both blanks with *B*
5.	a. $\sqrt{81}$ b. $\sqrt[3]{64}$ c. $\sqrt[4]{81}$ d. $\sqrt[5]{1}$
6.	a. 8 b. 27 c. 64 d. 125
7.	a. 2 b. 3 c. 4 d. 5
8.	a. 16 b. 81 c. 256 d. 625
9.	a. 2 b. 3 c. 4 d. 5
10.	a. 9 b. 4 c. 3 d. 1
11.	a. 3 b. 2 c. 1 d. 0
12.	a. 4 b. −2 c. −4 d. not possible
13.	You can check that your square root is accurate by squaring your result. Based on what you have learned so far, it is not possible for a number to be negative after it has been squared.
14.	a. $\frac{4}{9}$ b. $\frac{8}{27}$ c. $\frac{4}{9}$ d. $-\frac{8}{27}$
15.	a. $-\frac{3}{5}\cdot-\frac{3}{5}\cdot-\frac{3}{5}\rightarrow-\frac{27}{125}$ b. $\frac{x^8}{10{,}000}$ c. $\frac{36x^2}{121}$ d. $-\frac{32}{x^5y^{15}}$
16.	$-\frac{2}{3}$
17.	a. $\frac{2}{3}$ b. $-\frac{1}{2}$ c. $-\frac{4}{5}$ d. not possible
18.	You can check that your square root is accurate by squaring your result. Based on what you have learned so far, it is not possible for a number to be negative after it has been squared.
19.	$\sqrt{10}$; cannot simplify this more
20.	90

21.	a. $\frac{4}{20}\rightarrow\frac{1}{5}$ b. $\frac{6}{42}\rightarrow\frac{1}{7}$ c. $\frac{8}{72}\rightarrow\frac{1}{9}$
22.	a. ≈6.1 b. ≈7.7 c. ≈9.9
23.	a. 6.083 b. 7.746 c. 9.899
24.	No
25.	Yes. For example, $7=\frac{7}{1}$.
26.	a. $\frac{1}{2}$ b. $\frac{1}{4}$ c. $\frac{1}{5}$ d. $\frac{3}{4}$
27.	a. $\frac{1}{3}$ b. $\frac{1}{9}$ c. $\frac{2}{3}$ d. $\frac{5}{9}$
28.	a. 3.14 → 3.1415926... b. Calculators will vary → 6.08276253...
29.	Circle $\sqrt{3}$, $\sqrt{8}$, and $\sqrt{15}$.
30.	90
31.	a. 6 c. 10 e. 20 g. 12 b. 6 d. 10 f. 20 h. 12
32.	a. 8 c. 70 e. 48 g. 45 b. 8 d. 70 f. 48 h. 45
33.	a. $\sqrt{A}\cdot\sqrt{B}$ b. $\sqrt{X\cdot Y}$
34.	$5\sqrt{3}$
35.	$2\sqrt{5}$
36.	a. five root three b. seven root six c. four root ten
37.	a. $2\sqrt{3}$ b. $3\sqrt{2}$ c. $5\sqrt{2}$ d. $\sqrt{10}$
38.	a. $3\sqrt{3}$ b. 3 c. two
39.	a. $3\sqrt{2}$ b. $\sqrt{16}\sqrt{3}\rightarrow4\sqrt{3}$ c. $\sqrt{4}\sqrt{5}\rightarrow2\sqrt{5}$
40.	1, 4, 9, 16, 25, 36, 49, 64, 81, 100
41.	a. 16 b. 9 c. 100 d. 64
42.	a. 4 and 16 b. 4 and 16 c. 4, 9 and 36 d. 4, 9, 16, 36 and 144
43.	a. $\sqrt{4}\sqrt{5}\rightarrow2\sqrt{5}$ b. $\sqrt{4}\sqrt{6}\rightarrow2\sqrt{6}$ c. $\sqrt{4}\sqrt{7}\rightarrow2\sqrt{7}$
44.	a. $\sqrt{9}\sqrt{5}\rightarrow3\sqrt{5}$ b. $\sqrt{9}\sqrt{6}\rightarrow3\sqrt{6}$

	c. $\sqrt{9}\sqrt{7} \to 3\sqrt{7}$
45.	a. $\sqrt{25}\sqrt{2} \to 5\sqrt{2}$ b. $\sqrt{25}\sqrt{3} \to 5\sqrt{3}$ c. $\sqrt{22}$
46.	a. $\sqrt{49}\sqrt{2} \to 7\sqrt{2}$ b. $\sqrt{4}\sqrt{10} \to 2\sqrt{10}$
47.	a. $\sqrt{81}\sqrt{2} \to 9\sqrt{2}$ b. $\sqrt{4}\sqrt{26} \to 2\sqrt{26}$
48.	8
49.	99
50.	a. $2 \cdot 2\sqrt{2} \to 4\sqrt{2}$ b. $3 \cdot 2\sqrt{3} \to 6\sqrt{3}$
51.	a. $7 \cdot \sqrt{16}\sqrt{2} \to 7 \cdot 4\sqrt{2} \to 28\sqrt{2}$ b. $5 \cdot 3 = 15$ c. $2\sqrt{14}$
52.	a. 1 b. 25 c. 8
53.	H^2
54.	L^2
55.	a. subtracting 3.1 b. multiplying by 3 c. squaring an expression (raising an expression to the second power)
56.	Raise 10 to the 2nd power (compute 10^2)
57.	$z - 2 = P^2$ and $z = P^2 + 2$
58.	a. 17 b. 0 c. 2
59.	no solution
60.	a. 101 b. 6 c. 8 d. 24
61.	a. 27 b. −3 c. no solution
62.	a. $(x+2)(x+2) \to x^2 + 4x + 4$ b. $(x-1)(x-1) \to x^2 - 2x + 1$ c. $x^2 + 10x + 25$ d. $9x^2 - 42x + 49$
63.	a. $(x+3)(x+2) = 0 \to x = -3$ or -2 b. $(x-6)(x+3) = 0 \to x = 6, -3$ c. $(2x+3)(x-2) = 0 \to x = -1.5, 2$
64.	a. $0 = x^2 - 10x + 16 \to 0 = (x-8)(x-2)$ $\to x = 8$ (x = 8 or 2, but x = 2 makes the original equation false) b. $0 = x^2 + 9x + 8 \to 0 = (x+8)(x+1)$ $\to x = -1$ (x = −8 or −1, but x = −8 makes the original equation false)
65.	a. $\left(\sqrt{x+3}\right)^2 = (x+1)^2 \to x + 3 = x^2 + 2x + 1$ $\to 0 = x^2 + x - 2 \to 0 = (x+2)(x-1)$ $x = -2$ or 1 b. $x = 4$ or 3
66.	a. 1 is a solution, but −2 is **not** because it makes the original equation false b. 4 and 3 are both solutions
67.	$x = -1$ (x = −9 or −1, but −9 makes the original equation false)
68.	a. x = 8 (x = 3 or 8, but x = 3 makes the original equation false) b. no solution (x = −1, but that value makes the original equation false)
69.	no solution (x = 1 or 4, but both make the original equation false)
70.	N is 36
71.	a. $\sqrt{12} \to 2\sqrt{3}$ b. 5 c. 10 d. $2\sqrt{2} \cdot \sqrt{14} \to 2\sqrt{28} \to 2 \cdot 2\sqrt{7} \to 4\sqrt{7}$
72.	a. 24 b. $6x$ c. $6\sqrt{2}$ d. $2A\sqrt{B}$
73.	a. $20\sqrt{3}$ b. 12 c. $6\sqrt{8} \to 12\sqrt{2}$
74.	a. $4 \cdot 3\sqrt{3} \to 12\sqrt{3}$ b. $2\sqrt{16} \to 8$
75.	a. 48 b. $15xy$ c. $15\sqrt{6}$
76.	a. $44\sqrt{14}$ b. $AC\sqrt{BD}$ c. $4\sqrt{4} \to 8$
77.	a. $40\sqrt{2}$ b. $-30\sqrt{3}$ c. −36 d. 63
78.	$\sqrt{18} = 3\sqrt{2} \to 3(1.41) = 4.23$
79.	$\sqrt{75} = 5\sqrt{3} \to 5(1.73) = 8.65$
80.	a. $x = 7$ b. $x = 23$ c. $x = -1$
81.	G = 24; H = 48
82.	a. $3\sqrt{2}, \sqrt{19}, 2\sqrt{5}$ $\left(\sqrt{18}, \sqrt{19}, \sqrt{20}\right)$ b. $3\sqrt{10}, 7\sqrt{2}, 6\sqrt{3}, 4\sqrt{7}$ $\left(\sqrt{90}, \sqrt{98}, \sqrt{108}, \sqrt{112}\right)$
83.	a. $\sqrt{5}$ c. $2\sqrt{2}$ e. 4 b. 3 d. 2 f. $2 + 2\sqrt{3}$
84.	No. $2 \neq \sqrt{2}$
85.	a. 5 c. $5\sqrt{3}$ e. 6 b. 7 d. 5 f. 2
86.	a. No b. No
87.	a. $6xy$ b. $8MN$ c. $30x^3y^3$
88.	a. $\sqrt{4} = 2$. Thus, $\sqrt{4} + \sqrt{4} = 4$. b. $\sqrt{4} = 2$; $\sqrt{9} = 3$. Thus, $\sqrt{4} + \sqrt{9} = 5$. c. $\sqrt{36} = 6$; $\sqrt{25} = 5$. Thus, $\sqrt{36} - \sqrt{25} = 1$.
89.	a. 5 eggs b. $5x$ c. $5x^2$ d. $5\sqrt{2}$
90.	a. $2\sqrt{2}$ b. $2\sqrt{3}$ c. $3\sqrt{5}$
91.	a. $2\sqrt{2}$ b. $3\sqrt{3}$ c. $-4\sqrt{5}$

